**In Noisy Equations, One Who Heard Music**

Experts say Martin Hairer’s epic masterpiece in stochastic analysis “created a whole world.”

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Martin Hairer’s masterwork is so fantastic, so fully baked and so far out of left field, one fellow mathematician declared, that the manuscript must have been downloaded into his brain by a more intelligent alien race.

Another compared the 180-page treatise to the “Lord of the Rings” trilogy because it “created a whole world.” Few could recall another time in modern history when such a magnificent theory had emerged predominantly from the mind of a single person.

As for the researchers who have striven for decades to fathom the strange equations addressed by his theory, “He’s taken them all to the cleaners,” said [Terry Lyons](http://www.maths.ox.ac.uk/people/profiles/terry.lyons), a mathematician at the University of Oxford in England.

The [Tolkienesque paper](http://arxiv.org/abs/1303.5113), published online in the journal Inventiones Mathematicae in March, is only the latest and greatest in a series of feats by the 38-year-old [Hairer](http://www2.warwick.ac.uk/fac/sci/maths/people/staff/martin_hairer/), who has frequently stunned colleagues with the speed and creativity of his work. But if you were to take a seat next to Hairer at the pub near his home in Kenilworth, England, you might have a nice chat with him without ever suspecting the gangly, genial Austrian is one of the world’s most brilliant mathematicians.

“Martin likes to talk to people; people like to talk to him,” said his wife, the mathematician [Xue-Mei Li](http://www2.warwick.ac.uk/fac/sci/maths/people/staff/xue_mei_li/). He is good-natured, knowledgeable and calm, she said — “and funny enough.”

Today, Hairer is one of four recipients of the 2014 Fields Medal, announced on the [International Mathematical Union’s website](http://www.mathunion.org/general/prizes/2014/prize-citations/) and presented at the opening ceremony of the International Congress of Mathematicians in Seoul, South Korea. The Fields is widely viewed as the highest honor a mathematician can receive. Hairer, a professor at the University of Warwick in England, has been regarded since his late 20s as a leading figure in stochastic analysis, the branch of mathematics dealing with random processes like crystal growth and the spread of water in a napkin. Hairer’s colleagues particularly note his rare mathematical intuition, an ability to sense the way toward grand solutions and beautiful proofs.

Hairer (second from right); his wife, Xue-Mei Li (far right); and several colleagues from the University of Warwick gathered at the Virgins and Castle gastropub in Kenilworth.

“If you leave him alone for a couple of days, he comes back with a miracle,” said [Hendrik Weber](http://www2.warwick.ac.uk/fac/sci/maths/people/staff/hendrik_weber/), a colleague and collaborator at Warwick.

But, friends and colleagues say, this miracle worker’s talents coexist with a disarmingly down-to-earth nature, extracurricular interests and even an entire career outside of mathematics. A lover of rock music and computer programming, Hairer is the creator of an award-winning sound-editing program called [Amadeus](http://www.hairersoft.com/index.html), a popular tool among deejays, music producers and gaming companies and a lucrative sideline for Hairer.

“I don’t think he has any of the stereotypes that the general public would like to assign to a mathematician,” said [Ofer Zeitouni](http://www.wisdom.weizmann.ac.il/~zeitouni/), a professor of mathematics at the Weizmann Institute of Science in Israel who is introducing Hairer’s work today in Seoul.

Hairer’s well-roundedness has proven beneficial in a field that can seem detached from reality. It was his knowledge of a signal compression technique used in audio and image processing that inspired his otherworldly new theory.

The theory provides both the tools and the instruction manual for solving a huge class of previously unfathomable equations, statements that amount to “basically, ‘infinity equals infinity,’ ” in one specialist’s words, but which, despite their seeming senselessness, arise frequently in physics. The equations are mathematical abstractions of growth, the hustle and bustle of elementary particles and other “stochastic” processes, which evolve amid environmental noise.

It was these stochastic partial differential equations (SPDEs) that lured Hairer away from the path to a career as a physicist.

“It was just intriguing to me that you could derive these equations that don’t make sense,” he said in March during a stay at the Institute for Advanced Study in Princeton, N.J.

The mathematical inscrutability of many SPDEs has tantalized for decades. Their variables zigzag wildly through space and time, creating a mathematician’s nightmare, a corner, at every point; worse still, to solve the equations, the infinite sharpness of those corners must somehow be multiplied and otherwise manipulated. In some cases, physicists have found tricks for approximating the solutions to the equations, such as ignoring the infinite jaggedness of the curves below a certain scale. But mathematicians have long sought a more rigorous understanding.

“The thing I’ve been working on is to give meaning to the equations,” Hairer explained.

Hairer’s theory of “regularity structures” brings order to SPDEs by broadening many of math’s most basic concepts: derivatives, expansions and even what it means to be a solution. It is “kind of an extension of the classical calculus to this new setting,” said [Lorenzo Zambotti](http://www.proba.jussieu.fr/dw/doku.php?id=users:zambotti:index), a professor of mathematics at Pierre and Marie Curie University in Paris. Zambotti made the “Lord of the Rings” comparison upon reading a preliminary version of Hairer’s paper in early 2013, and has been studying the work ever since.

Experts describe Hairer’s paper as at once clear and dense, a tightly woven exposition that other mathematicians will need time to unravel but which will probably be used for decades or centuries to come.

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“Everyone knows it’s brilliant,” said [David Kelly](http://www.dtbkelly.com/), a mathematician at New York University and a former Ph.D. student of Hairer’s, “but everyone’s also quite scared of it.”

**A Logical Mind**

A study in contrasts, Hairer is tall — 6 feet 4 inches — but unassuming, with an angular face framing timid doe-eyes and a composure that is broken by frequent eruptions of hearty, boyish laughter. Hairer’s list of accomplishments —especially the latest entry — may be intimidating, but he is not. “He’s one of the least arrogant people I have ever met in my life,” Weber said.

Although he considers mathematics his primary interest, Hairer “likes to shut it off” and says he gets many of his best ideas while contemplating other things. Even while scrawling equations on dry erase boards and zooming in and out of infinitely jagged lines on his laptop screen to explain his work, he switches with ease to casual, friendly conversation.

Outside of math, he enjoys reading StephenKing thrillers and other “silly books,” cooking Asian-Western fusion dishes, skiing and going on frequent rambles through the countryside with Li. The two live in a semidetached, Victorian house in Kenilworth and sometimes ride their bikes together the few miles to and from the Warwick campus.

According to Li, it is Hairer’s diverse interests that inform his searing mathematical intuition, while his programming skills enable him to quickly test new ideas with algorithms. In her opinion, even his calm manner contributes to his success. “For large projects, people get overwhelmed, but he doesn’t,” she said. “He’s good.”

Behind Hairer’s “normal personality,” as many acquaintances put it, lies an uncommonly logical and organized mind. “Everything he learns he stores in an extremely structured way,” Kelly said. As a graduate student, Kelly often stopped by Hairer’s office to ask questions about stochastic analysis. “He would sort of stare off into the distance for 10 seconds and think about the question,” Kelly said, “and then grab a piece of paper and deliver a textbook-standard answer in response — three pages of extremely detailed notes.”

Hairer learned that he had won the Fields Medal during a visit to Columbia University in February. “It’s a big responsibility,” he said a month later. “You sort of become an ambassador of mathematics.”

Hairer said he did not expect to win and doesn’t see himself as a typical Fields medalist. For starters, by his own reckoning, he was not a child prodigy, although he was clearly a very smart kid. Born into an Austrian family living in Switzerland, Hairer spent most of his childhood in Geneva, where his father, [Ernst Hairer](http://www.unige.ch/~hairer/), works as a mathematician at the University of Geneva. Martin Hairer read chapter books by the precocious age of 6; became fluent in German, French and English; and performed at the top of his class all through school. “He was interested in everything,” Ernst Hairer recalled.

For his 12th birthday, in 1987, Martin’s father bought him a pocket calculator that could execute simple, 26-variable programs. He was instantly hooked. The next year, he persuaded his younger brother and sister to go in with him on a joint birthday present: a Macintosh II. He quickly became a proficient programmer, creating visualizations of fractals like the Mandelbrot set and then, at age 14, developing a program for solving ordinary differential equations — the much simpler cousins of SPDEs.

*Courtesy of Martin Hairer*

At 15, Hairer won a top prize at the 1991 European Community (now Union) Contest for Young Scientists in Zurich, Switzerland, with a computer interface for building electronic circuits.

With his program, Hairer advanced to the national level of the European Union Contest for Young Scientists. The next year, he won a prize at the highest — the European — level of the contest with an interface for designing and simulating electrical circuits. At 16, in his final year of eligibility for the contest, he was interested in the physics of sound as well as Pink Floyd and The Beatles. He enjoyed recording musical notes and looking at the resulting waveforms on his computer and tried to write a program that could extract the notes from the recordings. The task was too difficult, but he ended up with a program for manipulating the recordings: version 1 of Amadeus. The software was selected for the European level of the competition, but the judges did not allow Hairer to advance a second time.

Pulled in different directions by mathematics, physics and computer science, Hairer only settled on math in his early 20s. At the time, he was working on a research project involving SPDEs as a Ph.D. student in physics at the University of Geneva. The mathematical aspects of the equations struck him as far more compelling than the physical phenomena they described. The equations seemed to possess a hidden meaning. After all, physicists had many black magic tricks for making their calculations work, which seemed to miraculously transform the equations into surprisingly close models of reality. But mathematically, they were ill-defined.

“Physicists are very good at being able to extract actual information from equations without bothering about whether they actually have a meaning,” Hairer said, laughing. “They usually get it right, which is an amazing thing. But mathematicians like to actually know what the objects are.”

He also relished the idea that if he succeeded in developing a mathematical theory of SPDEs, his discovery would hold forever.

“One advantage of mathematics is the immortality,” he said. “Theorems that were proven 2,000 years ago are still true, whereas the physical worldview from 2,000 years ago definitely isn’t.”

Consider, for example, the divergent fates of Euclidean geometry — an ancient but enduring mathematical description of the architecture of flat space — and Aristotle’s celestial spheres — imaginary concentric shells centered on the Earth that were thought to physically rotate the stars and planets through the heavens. “In physics, I could probably defend the reasoning behind a theory,” Hairer said, “but I wouldn’t defend it with my life.”

**Regularity in Randomness**

Hairer and Li have worked at Warwick University for 11 and 7 years, respectively, though never on a joint project. They met at a conference in 2001, also at Warwick, while he was a graduate student at Geneva and she was a professor at the University of Connecticut. “I enjoyed talking to him from the very beginning,” said Li, who is originally from China. “I like the way he thinks and talks. Maybe I’m biased.” After a few years of academic shuffling, the couple settled in Warwick, a collaborative and lively academic environment that suits them both.

As Hairer’s career progressed, his talents became readily apparent, and according to experts in stochastic analysis, he has been “universally” renowned in the field for a decade.

His first major discovery came in 2004. Several groups were competing to prove that the two-dimensional stochastic Navier-Stokes equations — SPDEs that describe fluid flow in the presence of noise — are “ergodic,” or eventually evolve to the same average state independent of their initial inputs. While riding the train on his way to meet with his collaborator on the project, [Jonathan Mattingly](http://fds.duke.edu/db/aas/math/faculty/mattingly) of Duke University, Hairer had a sudden insight that grew into [a powerful, case-closing result](http://arxiv.org/pdf/math.PR/0406087.pdf).

In 2011, he solved a famous SPDE called the Kardar-Parisi-Zhang equation — a model of interface growth, such as the advancing edge of a bacterial colony in a petri dish and the spread of water in a napkin. The KPZ equation had been an open and much-studied problem since [physicists proposed it](http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.56.889) in 1986. Using an approach developed by Lyons called rough path theory, Hairer developed the core of his solution to the equation in less than a fortnight. Weber recalled that he went on vacation for 10 days right as Hairer began working. “I came back, and he had solved the whole thing and had already typed it up,” Weber said. “It was completely incredible to me.”

Hairer’s KPZ proof made a big splash, but by the time it [appeared in the Annals of Mathematics](http://arxiv.org/pdf/1109.6811v3.pdf) in summer 2012, he was already in the thick of developing a more sophisticated approach for solving the KPZ equation, as well as even more complex SPDEs: his magnum opus, the theory of regularity structures.

The problem with SPDEs is that they involve supremely thorny mathematical objects called “distributions.” When a water droplet suffuses a napkin, for example, the advance of the water’s edge depends on the current edge, as well as on noise: erratic factors like temperature variations and the creases and curves of the napkin. In the abstract form of an equation, the noise causes the edge to change infinitely quickly in space and time. And yet, according to the equation, the distribution that describes how quickly the edge changes in time is related to the square of the distribution describing how quickly it changes in space. But while smooth curves can easily be squared or divided, distributions do not submit to these arithmetic operations. “No object in the equations makes any sense classically,” said [Jeremy Quastel](http://www.math.toronto.edu/quastel/), a professor of mathematics at the University of Toronto.

For decades, mathematicians strove for a rigorous method of operating on distributions in order to solve SPDEs but made little headway. There are even published books that present incorrect procedures for doing so, Quastel said, “which is not something you would normally have in mathematics.”

Hairer’s big idea came to him in October 2011 while he was walking from the common room of the Warwick math department back to his office and thinking about nothing in particular. He suddenly realized that he could tame the distributions in SPDEs using an approach modeled on the mathematical properties of “wavelets” — brief, heartbeatlike oscillations that encode information in JPEG and MP3 files. Hairer had once considered using wavelets for a function in Amadeus. Conveniently for the purposes of data compression, any wavelet can be reconstructed by adding together a finite series of identical wavelets squashed to fractions of its initial width: a half, then a fourth, then an eighth and so on.

 simulation of the KPZ equation, which describes the stochastic growth of edges.

Similarly, Hairer realized, an infinitely jagged distribution like the ones that arise in SPDEs can also be written as a finite series. Each element of the series would consist of a set of curvelike objects that approximate the shape of the distribution at a fixed point in space and over a fixed time interval. In the next element in the series, this time interval would decrease to half, and then in the next to a fourth; as more elements in the series were included, the approximation would become more refined. Hairer suspected that, just as with wavelets, only a finite number of elements would be needed for the series to converge on the actual solution of the SPDE. If correct, he would be able to substitute the infinite and unfathomable distributions that arise in many SPDEs with a manageable number of perfectly calculable objects.

Right away, Hairer said, “I sort of knew it would work.”

He went home and told Li about his epiphany. They got a textbook off the shelf and looked up wavelets, as neither of them knew the object’s exact mathematical definition. Li soon saw that Hairer’s idea was brilliant. “I said he should pursue it and spend lots of time on it,” she said. “Instead of going out, just sit down and work.”

**Stochastic Future**

By the time Andrew Wiles proved Fermat’s Last Theorem in 1995, the 358-year-old problem had generated so much activity throughout its history that fame and recognition for Wiles were instantaneous. In Hairer’s case, no one reasonably expected a general theory of SPDEs. It seemed to come out of nowhere. “I suppose I kind of created a flurry of activity,” Hairer said.

Although dozens of mathematicians in Hairer’s immediate research area are learning his theory, some experts fear that it is too technically challenging to gain widespread use. “There is a worry that it won’t have the impact it deserves, not through any fault of Martin’s but just because the simplest way that one can deal with this type of problem is just too difficult to be popularizable,” said Quastel, who joked to colleagues that the theory must have been a dispatch from aliens. The power of regularity structures is easily understood, but when Hairer actually delves into how the theory works in talks and papers, Quastel said, “he loses his audience a little bit.”

If the theory does take hold, Lyons said, a deeper understanding of SPDEs could someday become useful in real physical models, such as in particle physics and machine learning. “There are an enormous number of situations where you have complex spatial behavior with randomness, where there is physical or social significance to understanding what the hell’s going on,” he said, “and I think Martin has made revolutionary contributions to our ability to tackle those things mathematically.”

The possibility that his theory might have physical relevance seems to hold little allure for Hairer, however. When asked whether he thought regularity structures might reveal something new about the “actual universe,” he merely laughed heartily. He said he finds joy in the newly discernible properties of the equations themselves, such as how far the solutions fluctuate away from their average value over long stretches of time or space or how rapidly two solutions with different starting points wriggle toward each other.

Picture these squirming solutions plotted side by side on the same graph — mathematical abstractions of, say, what would happen if water dripped onto two identical napkins.

“At some point they touch,” Hairer said, referring to such a pair of solutions. “You can ask how snugly do they fit each other at the point where they touch. And it turns out they are much snugger than you would think.” He laughed with delight.