**The Musical, Magical Number Theorist**

The search for artistic truth and beauty has led Manjul Bhargava to some of the most profound recent discoveries in number theory.
By: [Erica Klarreich](http://www.simonsfoundation.org/authors/erica-klarreich/?quanta)
August 12, 2014

For Manjul Bhargava, the counting numbers don’t simply line themselves up in a demure row. Instead, they take up positions in space — on the corners of a Rubik’s Cube, or the two-dimensional layout of the Sanskrit alphabet, or a pile of oranges brought home from the supermarket. And they move through time, in the rhythms of a Sanskrit poem or a tabla drumming sequence.

[Bhargava’s](https://www.math.princeton.edu/directory/manjul-bhargava) mathematical tastes, formed in his earliest days, are infused with music and poetry. He approaches all three realms with the same goal, he says: “to express truths about ourselves and the world around us.”

The soft-spoken, boyish mathematician could easily be mistaken for an undergraduate student. He projects a quiet friendliness that makes it easy to forget that the 40-year-old is widely considered one of the towering mathematical figures of his age. “He’s very unpretentious,” said [Benedict Gross](http://www.math.harvard.edu/~gross/), a mathematician at Harvard University who has known Bhargava since the latter’s undergraduate days. “He doesn’t make a big deal of himself.”

Yet the search for artistic truth and beauty has led Bhargava, a mathematics professor at Princeton University, to some of the most profound recent discoveries in number theory, the branch of mathematics that studies the relationships between whole numbers. In the past few years, he has made great strides toward understanding the range of possible solutions to equations known as elliptic curves, which have bedeviled number theorists for more than a century.

“His work is better than world-class,” said [Ken Ono](http://www.mathcs.emory.edu/~ono/), a number theorist at Emory University in Atlanta. “It’s epoch-making.”

Today, Bhargava was named one of this year’s four recipients of the Fields Medal, widely viewed as the highest honor in mathematics.

Bhargava “lives in a wonderful, ethereal world of music and art,” Gross said. “He floats above the normal concerns of daily life. All of us are in awe of the beauty of his work.”

Bhargava “has his own perspective that is remarkably simple compared to others,” said [Andrew Granville](http://www.dms.umontreal.ca/~andrew/), a number theorist at the University of Montreal. “Somehow, he extracts ideas that are completely new or are retwisted in a way that changes everything. But it all feels very natural and unforced — it’s as if he found the right way to think.”

**Musician**

Bhargava says that playing the tabla, a traditional Indian percussion instrument, and doing number theory research are both largely improvisational.

From early childhood, Bhargava displayed a remarkable mathematical intuition. “Teach me more math!” he would badger his mother, [Mira Bhargava](http://www.hofstra.edu/Faculty/fac_profiles.cfm?id=114&t=/Academics/Colleges/HCLAS/MATH/), a mathematics professor at Hofstra University in Hempstead, N.Y. When he was 3 years old and a typical, rambunctious toddler, his mother found that the best way to keep him from bouncing off the walls was to ask him to add or multiply large numbers.

“It was the only way I could make him stay still,” she recalled. “Instead of using paper and pencil, he would kind of flip his fingers back and forth and then give me the right answer. I always wondered how he did it, but he wouldn’t tell me. Perhaps it was too intuitive to explain.”

Bhargava saw mathematics everywhere he looked. At age 8, he became curious about the oranges he would stack into pyramids before they went into the family juicer. Could there be a general formula for the number of oranges in such a pyramid? After wrestling with this question for several months, he figured it out: If a side of a triangular pyramid has length *n*, the number of oranges in the pyramid is *n*(*n*+1)(*n*+2)/6. “That was an exciting moment for me,” he said. “I loved the predictive power of mathematics.”

Bhargava quickly became bored with school and started asking his mother if he could go to work with her instead. “She was always very cool about it,” he recalled. Bhargava explored the university library and went for walks in the arboretum. And, of course, he attended his mother’s college-level math classes. In her probability class, the 8-year-old would correct his mother if she made a mistake. “The students really enjoyed that,” Mira Bhargava said.

Every few years, Bhargava’s mother took him to visit his grandparents in Jaipur, India. His grandfather, Purushottam Lal Bhargava, was the head of the Sanskrit department of the University of Rajasthan, and Manjul Bhargava grew up reading ancient mathematics and Sanskrit poetry texts.

To his delight, he discovered that the rhythms of Sanskrit poetry are highly mathematical. Bhargava is fond of explaining to his students that the ancient Sanskrit poets figured out the number of different rhythms with a given number of beats that can be constructed using combinations of long and short syllables: It’s the corresponding number in what Western mathematicians call the Fibonacci sequence. Even the Sanskrit alphabet has an inherent mathematical structure, Bhargava discovered: Its first 25 consonants form a 5 by 5 array in which one dimension specifies the bodily organ where the sound originates and the other dimension specifies a quality of modulation. “The mathematical aspect excited me,” he said.

**Fibonacci Rhythms**

Sanskrit poems feature a mix of short and long syllables that last for one or two beats, respectively. As a child, Bhargava was fascinated by the question of how many different rhythms it is possible to construct with a given number of beats. A four-beat phrase, for example, could be short-long-short or short-short-short-short (or one of three other possibilities).

The answer, Bhargava discovered, was given in “Chandahsastra,” a treatise on poetic rhythms written by Pingala more than two millennia ago. There’s a simple formula: The number of different rhythms with, say, nine beats is the sum of the number of rhythms with seven beats and the number of rhythms with eight beats. That’s because each nine-beat rhythm can be constructed by adding either a long syllable to a seven-beat rhythm or a short syllable to an eight-beat rhythm.

This rule generates the sequence 1, 2, 3, 5, 8, 13, 21, 34, 55, and so on —in which each number is the sum of the preceding two. These are known as the Hemachandra numbers — after 11th-century scholar Acharya Hemachandra, who wrote about poetic rhythms — or the Fibonacci numbers, to Western mathematicians. Bhargava enjoys showing his students that these numbers arise not only in poetic rhythms but also in natural settings, such as in the number of spirals on a pine cone or petals on a daisy.

At Bhargava’s request, his mother started teaching him to play tabla**,** a percussion instrument of two hand drums**,** when he was 3 (he also plays the sitar, guitar and violin). “I liked the intricacy of the rhythms,” he said, which are closely related to the rhythms in Sanskrit poetry. Bhargava eventually became an accomplished player, even studying tabla with the legendary [Zakir Hussain](http://www.zakirhussain.com/) in California. He has performed in concert halls around the country and even at Central Park in New York City.

“He’s a terrific musician who has reached a very high technical level,” said [Daniel Trueman](http://www.princeton.edu/music/people/display_person.xml?netid=dtrueman&display=Faculty), a music professor at Princeton who collaborated with Bhargava on a performance over the Internet with musicians in Montreal. Just as important, he said, is Bhargava’s warmth and openness. Even though Trueman’s background is not primarily in Indian music, “I never felt that I was offending his high level of knowledge of North Indian classical music,” Trueman said.

Bhargava often turns to the tabla when he is stuck on a mathematics problem, and vice versa. “When I go back, my mind has cleared,” he said.

He experiences playing the tabla and doing mathematics research similarly, he said. Indian classical music — like number theory research — is largely improvisational. “There’s some problem-solving, but you’re also trying to say something artistic,” he said. “It’s similar to math — you have to put together a sequence of ideas that enlightens you.”

Mathematics, music and poetry together feel like a very complete experience, Bhargava said. “All kinds of creative thoughts come together when I think about all three.”

**Mathematician**

Between attending his mother’s classes and traveling to India, Bhargava missed a lot of school over the years. But on the days he didn’t go to school, he would often meet his schoolmates in the afternoon to play tennis and basketball. Despite his extraordinary intelligence, “he was just a normal kid, associating with all the kids,” Mira Bhargava recalled. “They were completely at ease with him.”

That’s a refrain repeated by Bhargava’s colleagues, students and fellow musicians, who describe him using words like “sweet,” “charming,” “unassuming,” “humble” and “approachable.” Bhargava wears his mathematical superstardom lightly, said Hidayat Husain Khan, a professional sitarist based in Princeton and India who has performed with him. “He has the ability to connect with a huge spectrum of people, regardless of their background.”

The only time that Bhargava’s extended school absences threatened to harm him was when his high school health teacher tried to block him from graduating — even though he was the valedictorian and had been accepted to Harvard. (He did graduate.)

It was at Harvard that Bhargava decided, once and for all, to pursue a career in mathematics. With such eclectic interests, he had flirted with many possible careers — musician, economist, linguist, even mountain climber. Eventually, however, he realized that it was usually the mathematical aspects of these subjects that got him most excited.

“Somehow, I always came back to math,” he said.

Bhargava felt the strongest tug between mathematics and music but decided in the end that it would be easier to be a mathematician who did music on the side than a musician who did mathematics on the side. “In academia, you can pursue your passions,” he said.

Zometools are just one of the many math toys that decorate Bhargava’s 12th floor office at Princeton University.

Now, Bhargava has an office on the 12th floor of Princeton’s Fine Hall littered with math toys — Rubik’s Cubes, Zometools, pine cones and puzzles. When he is thinking about mathematics, however, Bhargava prefers to escape his office and wander in the woods. “Most of the time when I’m doing math, it’s going on in my head,” he said. “It’s inspirational being in nature.”

This approach can have its drawbacks: More than once, Bhargava has postponed writing down an idea for years only to forget the specifics. At times, however, delays between thinking and writing are inevitable. “Sometimes, when I have a new idea, there hasn’t been language developed to express it yet,” he said. “Sometimes, it’s just a picture in my mind of how things should flow.”

Although Bhargava uses his office primarily for meetings, the mathematical toys decorating its surfaces are more than just a colorful backdrop. When he was a graduate student at Princeton, they helped him solve a 200-year-old problem in number theory.

If two numbers that are each the sum of two perfect squares are multiplied together, the resulting number will also be the sum of two perfect squares (Try it!). As a child, Bhargava read in one of his grandfather’s Sanskrit manuscripts about a generalization of this fact, developed in the year 628 by the great Indian mathematician Brahmagupta: If two numbers that are each the sum of a perfect square and a given whole number times a perfect square are multiplied together, the product will again be the sum of a perfect square and that whole number times another perfect square. “When I saw this math in my grandfather’s manuscript, I got very excited,” Bhargava said.

There are many other such relationships, in which numbers that can be expressed in a particular form can be multiplied together to produce a number with another particular form (sometimes the same form and sometimes a different one). As a graduate student, Bhargava discovered that in 1801, the German mathematical giant Carl Friedrich Gauss came up with a complete description of these kinds of relationships if the numbers can be expressed in what are known as binary quadratic forms: expressions with two variables and only quadratic terms, such as *x*2 + *y*2 (the sum of two squares), *x*2 + 7*y*2, or 3*x*2 + 4*xy* + 9*y*2. Multiply two such expressions together, and Gauss’ “composition law” tells you which quadratic form you will end up with. The only trouble is that Gauss’ law is a mathematical behemoth, which took him about 20 pages to describe.

Bhargava wondered whether there was a simple way to describe what was going on and whether there were analogous laws for expressions involving higher exponents. He has always been drawn, he said, to questions like this one — “problems that are easy to state, and when you hear them, you think they’re somehow so fundamental that we have to know the answer.”

The answer came to him late one night as he was pondering the problem in his room, which was strewn with Rubik’s Cubes and related puzzles, including the Rubik’s mini-cube, which has only four squares on each face. Bhargava — who used to be able to solve the Rubik’s Cube in about a minute — realized that if he were to place numbers on each corner of the mini-cube and then cut the cube in half, the eight corner numbers could be combined in a natural way to produce a binary quadratic form.

There are three ways to cut a cube in half — making a front-back, left-right or top-bottom division — so the cube generated three quadratic forms. These three forms, Bhargava discovered, add up to zero — not with respect to normal addition, but with respect to Gauss’ method for composing quadratic forms. Bhargava’s cube-slicing method gave a new and elegant reformulation of Gauss’ 20-page law.

Additionally, Bhargava realized that if he arranged numbers on a Rubik’s Domino — a 2x3x3 puzzle — he could produce a composition law for cubic forms, ones whose exponents are three. Over the next few years, Bhargava [discovered](http://www.jstor.org/stable/3597249) [12 more](http://www.jstor.org/stable/3597310) composition laws, which formed the core of his Ph.D. thesis. These laws are not just idle curiosities: They connect to a fundamental object in modern number theory called an ideal class group, which measures how many ways a number can be factored into primes in more complicated number systems than the whole numbers.

“His Ph.D. thesis was phenomenal,” Gross said. “It was the first major contribution to Gauss’ theory of composition of binary forms for 200 years.”

**Magician**

Bhargava’s doctoral research earned him a five-year Clay Postdoctoral Fellowship, awarded by the [Clay Mathematics Institute](http://www.claymath.org/) in Providence, R.I., to new Ph.D.s who show leadership potential in mathematics research. He used the fellowship to spend one additional year at Princeton and the neighboring Institute for Advanced Study and then moved to Harvard. Only two years into his fellowship, however, job offers started pouring in, and a bidding war soon erupted over the young mathematician. “It was a crazy time,” Bhargava said. At 28, he accepted a position at Princeton, becoming the second-youngest full professor in the university’s history.

Back at Princeton, Bhargava felt like a graduate student again and had to be reminded by his former professors that he should call them by their first names now. “That was a little weird,” he said. Bhargava ordered some frictionless chairs for his office, and he and his graduate student friends would race down the halls of Fine Hall in the evenings. “One time, another professor happened to be there in the evening, and he came out of his office,” Bhargava said. “That was rather embarrassing.”

“He has proven some of the most exciting theorems in the past 20 years of number theory. The questions he attacks sound like things he shouldn’t have the right to answer.”

Bhargava is glad to be at an institution where he has the opportunity to teach. As an undergraduate teaching assistant at Harvard, he won the [Derek C. Bok Award](http://bokcenter.harvard.edu/icb/icb.do?keyword=k1985&pageid=icb.page495198) for excellence in teaching three years running. He especially enjoys reaching out to students in the arts or humanities, some of whom may think of themselves as mathphobic. “Because I came to math through the arts, it has been a passion of mine to bring in people who think of themselves as more on the art side than the science side,” he said. Over the years, Bhargava has taught classes on the mathematics of music, poetry and magic. “I think anyone is reachable if the material is presented in the right way,” he said.

Carolyn Chen, a Princeton undergraduate who took Bhargava’s freshman seminar on mathematics and magic, called the course “super chill.” Bhargava started each class by performing a magic trick — something he loves to do — and then the students dissected its mathematical principles. Bhargava’s colleagues had warned him to steer clear of proofs, he said, “but by the end of the course, everyone was coming up with proofs without realizing that’s what they were doing.”

The course inspired Chen and several classmates to take more proof-based mathematics classes. “I took number theory after that freshman seminar,” she said. “I would never have thought of taking it if I hadn’t taken his class, but I really enjoyed it.”

At Princeton, Bhargava started developing an arsenal of techniques for understanding the “geometry of numbers,” a field somewhat akin to his childhood orange counting that studies how many points on a lattice lie inside a given shape. If the shape is fairly round and compact, like a pyramid of oranges, the number of lattice points inside the shape corresponds approximately to the shape’s volume. But if the shape has long tentacles, it may capture many more — or many fewer — lattice points than a round shape of the same volume. Bhargava developed a way to understand the number of lattice points that appear in such tentacles.

“He has applied this method to one problem after another in number theory and just knocked them off,” Gross said. “It’s a beautiful thing to watch.”

While Bhargava’s early work on composition laws was a solo flight, much of his subsequent research has been in collaboration with others, something he describes as “a joyous experience.” Working with Bhargava can be intense: At times, said Xiaoheng Wang, a postdoctoral researcher at Princeton, he and Bhargava have begun discussing a math problem, and the next thing he knows, seven hours have passed. Characteristically, Bhargava is quick to deflect the honor of winning the Fields Medal onto his collaborators. “It’s as much theirs as mine,” he said.

In recent years, Bhargava has collaborated with several mathematicians to study elliptic curves, a type of equation whose highest exponent is three. Elliptic curves are one of the central objects in number theory: They were crucial to the proof of [Fermat’s Last Theorem](http://www-history.mcs.st-and.ac.uk/HistTopics/Fermat%27s_last_theorem.html), for example, and also have applications in cryptography.

A fundamental problem is to understand when such an equation has solutions that are whole numbers or ratios of whole numbers (rational numbers). Mathematicians have long known that most elliptic curves have either one rational solution or infinitely many, but they couldn’t figure out, even after decades of trying, how many elliptic curves fall into each category. Now, Bhargava has [started to clear up this mystery](http://www.simonsfoundation.org/quanta/20130709-mathematicians-shed-light-on-minimalist-conjecture/). With [Arul Shankar](http://abel.harvard.edu/people/ShankarArul.html), his former doctoral student who is now a postdoc at Harvard, Bhargava has shown that [more than 20 percent of elliptic curves have exactly one rational solution](http://arxiv.org/pdf/1312.7859.pdf). And with [Christopher Skinner](https://www.math.princeton.edu/directory/christopher-skinner), a colleague at Princeton, and [Wei Zhang](http://www.math.columbia.edu/~wzhang/) of Columbia University, Bhargava [has shown](http://arxiv.org/pdf/1401.0233.pdf) that at least 20 percent of elliptic curves have an infinite set of rational solutions with a particular structure called “rank 1.”

Bhargava, Skinner and Zhang have also made progress toward proving the famous [Birch and Swinnerton-Dyer conjecture](http://www.claymath.org/millenium-problems/birch-and-swinnerton-dyer-conjecture), a related problem about elliptic curves for which the Clay Mathematics Institute has offered a million-dollar prize. [Bhargava, Skinner, and Zhang have shown](http://arxiv.org/pdf/1407.1826.pdf) that the conjecture is true for more than 66 percent of elliptic curves.

Bhargava’s work on elliptic curves “has opened a whole world,” Gross said. “Now everybody is excited about it and jumping in to work on it with him.”

“He has proven some of the most exciting theorems in the past 20 years of number theory,” Ono said. “The questions he attacks sound like things he shouldn’t have the right to answer.”

Bhargava has developed a unique mathematical style, Gross said. “You could look at a paper and say, ‘Manjul’s the only one who could have done that.’ It’s the mark of a really great mathematician that he doesn’t have to sign his work.”

[*Thomas Lin*](http://www.simonsfoundation.org/authors/thomas-lin/?quanta) *contributed reporting from Princeton, N.J.*