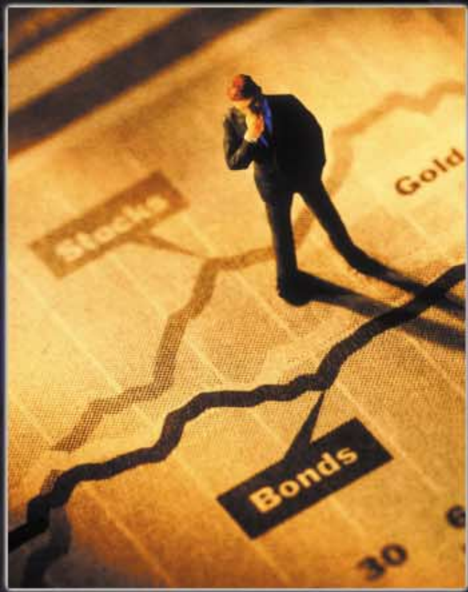
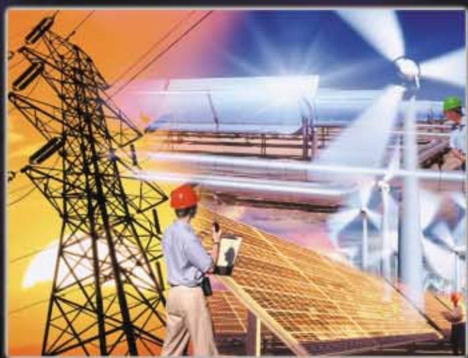
Mathematics:

Giving Industry the Edge





An industrial roadmap for mathematics and computing

Smith institute
for industrial mathematics and system engineering

Faraday Partnership

Designed and produced by Neil Buchan at the AEA Technology Imaging Centre

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Executive summary

Where are the opportunities for UK industry to gain a competitive edge through the exploitation of mathematics and computing? Over 50 top industrialists have now helped to answer this question. In this document, their vision of the challenges facing industry over the next 10 years has been matched to the expertise of the science base to produce the UK's first ever industrial roadmap for mathematics and computing.

Leading industrialists shaped the roadmap through a series of interviews, during which six scientific themes clearly emerged as being of prime importance to industrial competitiveness: **simulation**, **multidisciplinary design**, **uncertainty and risk**, **data**, **complex systems**, and **market behaviour**.

Interviewees were drawn from all sectors of the economy and more than 95% of the industrialists who were asked to contribute volunteered their time – testament in itself to the widespread recognition that the UK's mathematical resources have a vital role to play in wealth creation.

By connecting the six themes with the relevant mathematical and computational methods, the roadmap shows the ever-increasing potential for coupling industry and the science base to best mutual advantage. The roadmap reveals that there is a sharp contrast with the general pattern of collaboration between industrial and academic researchers in the past 30 years, which has been concentrated heavily in the theme of simulation.

The vision for the future is one of innovative collaborations, assembled to address the multi-thematic challenges of the modern economy and spanning a vastly wider range of mathematical and computational expertise than at present. These collaborations, supported through initiatives such as the Smith Institute, will create completely new mathematical frameworks for addressing the most important industrial challenges.

Mathematics is the most versatile of all the sciences. It is uniquely well placed to respond to the demands of a rapidly changing economic landscape. Just as in the past, the systematic application of mathematics and computing to the most challenging industrial problems will be a vital contributor to business performance. The difference now is that the academic community must broaden its view of mathematics in industry and its expertise must be managed in more imaginative ways.

Mathematics now has the opportunity more than ever before to underpin quantitative understanding of industrial strategy and processes across all sectors of business. Companies that take best advantage of this opportunity will gain a significant competitive advantage: mathematics truly gives industry the edge.

April 2002

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1 Introduction

Background to the roadmap

The purpose of this roadmap is to guide the strategic coupling of UK industry to the science base in mathematics and computing. It presents a vision for the application of mathematics and computing to the economic advantage of UK business and industry over the next 10 years. The roadmap is being made freely available and will be of interest to industrialists, academics and policy makers, in helping to widen the **opportunities for academics to contribute to industrial problems** and in showing where **industry can help to steer university research programmes**.

The roadmap has been prepared by the Smith Institute for Industrial Mathematics and System Engineering. As a Faraday Partnership, supported by the Department of Trade and Industry and the Engineering and Physical Sciences Research Council, the Smith Institute engages widely across industrial sectors and within the science base. As of April 2002, there are more than 80 companies and 25 universities contributing to its activities. The roadmap will help to develop further this strategic alignment between industry and academia.

Scope of the roadmap

The roadmap is deliberately based upon an analysis of industrial needs over the next ten years; in other words it is an **industrial roadmap, driven by industry 'pull'**. It has an emphasis different from technology roadmaps, which are driven by technology 'push'. Consequently, there are areas featuring strongly in current technology roadmaps that are not so prominent here, since they do not yet make up significant components of the economy. Some of these are currently the subject of heavy national and international research investment, such as bioinformatics and nanotechnology, and will eventually generate significant commercial activity. They would therefore be likely to feature strongly in future editions of the roadmap; indeed, the importance of mathematics and computation in bioinformatics is already clear.[1]

In addition, the roadmap restricts attention to *new* advances in the application of mathematics and computing. It is not concerned with situations where the relevant mathematical and computational expertise is *already* easily available through standard software tools, such as in the routine analysis of the structural performance of engineering components. There is clearly great value in such tools, since they allow engineers and designers who neither have nor need a deep understanding of the underpinning mathematics to benefit from the associated commercial potential. However, they are not a primary concern of the science base, and so lie outside the scope of this study.

[1] *UK Bioinformatics for Functional Genomics: Watching the Detectives*, Peter Grindrod, Numbercraft Limited, March 2001, www.maths.bath.ac.uk/~as/grindrod_mar2001.pdf

Structure of the roadmap

At the heart of the roadmap is an understanding of how different business sectors are likely to respond to their market drivers. The roadmap team conducted interviews with more than 50 senior industrialists, to clarify the technological challenges involved in meeting the demand for new products and processes. The exercise gathered data from sectors that are representative of the UK economy as a whole, based on standard stock market classifications.

From this information, six scientific themes have been identified that are of central importance in addressing the key challenges facing business and industry in today's knowledge-based economy. They are **simulation**, **multidisciplinary design**, **uncertainty and risk**, **data**, **complex systems**, and **market behaviour**. The roadmap connects these six themes to groups of mathematical and computational techniques. In doing so, it is demonstrated how mathematics and computing can underpin progress in key sectors of the economy. The themes serve as a hinge with which to connect industry to the science base in building new collaborations. Note here that the science base's expertise in mathematics and computing is found not only in university departments of mathematics, computing and statistics, but also in departments of engineering, economics, physics and other sciences.

This thematic approach to mathematics in industry is novel. It should be set against the historical background of figure 1, which is a catalogue of over 150 industrial applications of mathematics. These projects have been selected from documented collaborations world-wide over the last 30 years. They are grouped by industrial sector and colour-coded to indicate the themes involved. The most striking features of this catalogue are the **universal application of mathematics and computing across all sectors of the economy** and the historical dominance of the simulation theme. Simulation has been the natural outlet for core applied mathematics in the UK and many other countries; it is therefore more mature than the other themes and is where mathematics has traditionally had greatest industrial impact. The roadmap's vision for the future confirms the continuing role of simulation, while at the same time pointing clearly towards the rising importance of the other five themes.

Figure 1: Thirty years of mathematics in industry

The following topics have all been the subject of collaborations over the past 30 years; they are colour-coded to show the relevant roadmap themes.[1]

Simulation | **Data**
Multidisciplinary Design | **Complex Systems**
Uncertainty and Risk | **Market Behaviour**

Aerospace and defence
- Fluid loaded parallel plate wave guides
- Friction welding
- Modelling geophone arrays
- Modelling performance of a piezoelectric gyroscope
- Ship track reconstruction and data diffusion
- Stickslip oscillation in tailplane control systems
- Sun glitter in oceanic detection

Automotive
- Filtering combustion products in airbags
- Geometry of caravan awnings
- Optimal scheduling in automobile assembly
- Modelling catalytic converter performance
- Modelling of linered engine blocks
- Models for electrolytic painting in the car industry
- Sunroof boom

Banking, insurance and finance
- Efficient portfolio selection
- Financial bond sweeteners
- Long term foreign currency exchange rate predictions
- Options on currency baskets

Basic materials
- Bubble-bursting in molten glass
- Cold rolling of metal foil
- Cooling of steel by water sprays
- Die swell and blebbing
- Dip coating of lenses
- Edge corrugations in windscreens
- Edge profile in hot rolling
- Flux consumption in continuous casting
- Gravity currents in batch/flux flow in a glass furnace
- Heat transfer in a continuous bloomcaster
- High speed coating of optical fibres
- Impedance imaging in molten steel
- Injection moulding of zinalco key blanks
- Lens polishing on a rouge layer
- Modelling gas flow in tuyeres
- Moisture transport in drying cables
- Premature wearing of an oxygen lance
- Reduction of iron sand pellets
- Scanning vibrating needle curemeters
- Stratification in steel making ladles
- Stuffer box crimping
- Vacuum moulding of a superplastic
- Warping of moulded plastics

Chemical
- Backwash in microfiltration
- Current distribution in an array of spigots
- Fouling layers on heat exchangers
- Microwave heating of a flowing slurry
- Modulated differential scanning calorimetry
- Mudcracking in drying paint
- Periodic flow in porous regenerators
- Pinholing in screen printing
- Powder deposition from a charged cloud
- PVC autoclave models
- Three-state models for gel electrophoresis

Construction
- Air-conditioning system optimization
- Bending of plates on concrete foundations
- Effect of water injection to stabilize buildings
- Landslide-generated bores in reservoirs
- Light emission from high pressure sodium lamps
- Scheduling in the manufacture of evaporative air conditioners
- Stability of anchor cables
- Thermally pressing corner profiles

Electronics and computer hardware
- Beam control in electron beam lithography
- Free boundary models for p-i-n diodes
- Free boundary models for thermistors
- Ink drying in inkjet printers
- Magnetic recording and playback processes
- Magnetization reversal in particulate recording phenomena
- Singular stresses in semiconductor packaging
- Statistical optics and effective medium models of colour

[1] For further details of any topic, please contact office@smithinst.ac.uk

Figure 1: Thirty years of mathematics in industry (continued)

Energy and utilities
- Absorption of water wave energy by viscoelastic mats
- Behaviour of Bentonite clay in toxic waste barriers
- Bubbly liquid models in sewage pipes
- Combustion of coal in fluidized ash beds
- Contaminant transport in municipal water systems
- Current leakage from high tension cables to earth
- Downwind pollutant deposition from power stations
- Gas field scheduling
- Impact of grid frequency fluctuations
- Laser drilling
- Noise generation by water pipe leaks
- Oil blending, mixing and contamination
- Particle size in cyclone dust separators
- Rapid depressurization in liquid filled pipes
- Reduction of valve wear in pumps
- Risk assessment in highly skewed distributions
- Sea water percolation in a sand bank
- Stability of 3-phase boiler flow
- Stochastic model benefit analysis
- Turbulent combustion in confined spaces
- Vertical transformer oscillations under short circuit conditions
- Voids in fluidized beds
- Water splashing in cooling towers
- Whirling modes in nuclear fuel rod oscillations

Food and drink
- Cooling of jarred cheese spreads
- Decision support systems for beef stations
- Evaluation of fish freshness by pressure testing
- Optimal control of plate evaporators
- Optimal freezing of ice-lollipops
- Scheduling tirage champagne production
- Temperatures in cold rooms
- Thermal probe for food composition measurement
- Wet gum labelling of wine bottles

IT and software
- Interprocessor memory contention
- Scaling and optimization for list matching of agents
- Secure information flows in computer systems

Leisure and entertainment
- Data fusion using random conditional sets
- Deinterlacing TV images
- Enhancement in colour negative film development
- Mechanical characteristics of carbon fibre yacht masts
- Variable forgetting factors in Kalman filtering

Medicine
- Angiogenesis and tubule formation
- Classification of electromagnetic flow meters
- Detection of metastasis in human lungs from CT scans
- Modelling of a trickle bed bioreactor
- Signal analysis for heart pacemakers
- Telemetry problems with pacemakers in a noisy environment
- Waiting lists and theatre scheduling for surgical procedures

Mining
- Barriers for coal dust suppression
- Blending methodologies in talc production
- Cliff face disintegration in iron ore mining
- Confined detonation
- Diamond drilling for core sampling
- Dynamics of large mining excavations
- Electrode fracture
- Fault location in coal seams
- Foam break-up in cyclones
- Granular material models for bunker coal
- Moisture movement in bulk stockpiles
- Optimized drag line planning models
- Slurry behaviour in separation devices
- Strategic planning and optimization in an open coal mine

Paper and forestry
- Modelling of structural timber elements
- Paper stiffness sensing
- Pattern reduction in paper cutting
- Selection and mating decisions in tree breeding populations

Pharmaceutical
- Classification of pharmacore structures
- Drug diffusion through the skin

Retail and distribution
- Optimal customer account classification
- Scheduling and optimizing mineral train lengths

Telecommunications
- Fluid models for teletraffic
- Multimedia call centres
- Ray tracing for mobile phones
- Resource planning for optimum service quality
- Signal design and multichannel communication
- Spin glass models for associative memory
- Stochastics of slotted communication channels
- Throughput of ADSL modems
- UMTS network planning

Textiles
- Colour prediction in fluorescent dyed fabrics
- Drying of fibres in a gas blower
- Kinetic models of photobleaching
- Stability of air jet spinning of man-made fibres
- Thermal instability in man-made fibre spinning
- Tow washing
- Use of detergent in wool scouring

Transport
- Airline crew scheduling
- Aviation system capacity assessment tools
- Buckling of ship bulkheads
- Current collection systems for an electric locomotive
- Modelling of bicycle jams
- Optimal air traffic scheduling
- Pavement stresses due to tyre impact
- Sequencing aircraft departures
- Shock response of ship shafts
- Stability of ship autopilots
- Stiffness and damping in high speed tilting-pad bearings
- Surfing ship waves
- Wheel replacement and restoration in trains

2 Scientific themes

The six scientific themes that clearly emerged from the interviews with industry are:

- simulation
- data
- multidisciplinary design
- complex systems
- uncertainty and risk
- market behaviour

Each of these themes is present in the challenges facing many sectors of the economy and some apply to almost all sectors. The following sections explore the content of each theme in turn, drawing on evidence from the roadmap interviews to show how the theme arises in various industries. Each section concludes with a summary of the associated mathematical challenges. The overall linkage between sectors and themes is shown in table 1 on page 18.

2.1 Simulation

Simulation begins with a mathematical formulation, or 'model', of a real-world phenomenon. The model is then 'solved' using analytical and numerical techniques to produce quantitative predictions and understanding. Deficiencies in the simulation may arise either because the phenomenon being modelled is complex and poorly understood (i.e. the model is unreliable), or because the problem is computationally intractable (i.e. the available solution techniques are insufficient).

Simulation of physical and biological systems has become an essential component of modern science and engineering. With the widespread availability at low cost of ever more powerful computers, most sectors of UK industry now routinely use simulation to support, and sometimes to replace, experimentation. The further step of incorporating simulation into the design process creates additional benefit, especially when the product development cycle is costly and lengthy, such as in the aerospace and automotive industries.

Many examples of simulation were cited in the roadmap interviews.

- In the **aerospace** and **automotive** sectors, improving the efficiency of combustion processes reduces fuel consumption, controls harmful emissions and so is beneficial to the environment. Understanding the air flow over wings and fuselage is vital to the performance of aircraft and also to their safety against icing. Aerodynamic considerations are increasingly important in the design of cars, lorries and trains. Controlling the noise generated by all types of vehicles can improve passenger comfort and minimize noise pollution of the environment. Making sure that the electrical equipment in an aircraft or car functions correctly in the presence of electromagnetic fields, and studying how an aircraft might be affected by a lightning strike, are important design considerations that impact on performance and safety. Producing new materials capable of withstanding large forces and extremes of temperature leads to more reliable and efficient jet engines. Designing ships and aircraft so that they are harder to detect using radar is important for the **defence** industry. Physical simulation plays a vital role in all these activities.

- In the **food** industry, the final texture of baked goods, e.g. bread, cake and crumpets, depends upon how the original material (e.g. dough in the case of bread) evolves during baking. A better understanding of baking and other processes would enable the industry to be more efficient in the design of processes for new products and in responding to changes in raw ingredients. A separate application of simulation is in food-sorting devices, where the need is for fast devices with multiple uses.

- The **chemical** industry uses a range of simulation techniques, such as computational fluid dynamics and molecular models, to improve manufacturing processes and to design new products that are customized for particular applications. For example, the properties of an adhesive determine what it can be used for and how it must be applied.

- In the **construction** industry, large companies routinely use simulation to evaluate the effectiveness of lighting, heating and, in the case of theatres, acoustics. Deleterious effects due to wind, snow loading and vibration are also investigated. In safety-critical environments, such as airports, one also wishes to simulate the behaviour of humans in response to fire or terrorist attack.

- In the **oil and gas** industry, physical simulation is used to predict the flow of oil and gas in underground reservoirs, in order to maximize the amount of the resource extracted. It is also used to understand how oil and gas flow through the very long pipelines that are often needed to transport them.

- In the **pharmaceuticals** sector, simulation is used in the design of drug delivery systems. For example, asthma inhalers must be mechanically reliable and, despite slight manufacturing variations, capable of delivering a dose within a specified tolerance. To achieve this, both the stresses in the inhaler and the flow of the drug must be modelled. Moreover, simulation in the form of computational molecular biology is helping pharmaceutical companies to understand the action of drugs, and to design them for specific therapeutic purposes.

- In the **medical** sector, simulation is used to design patient-specific and longer lasting joint implants, thereby reducing the need for additional surgery. It is also used to study the behaviour of the soft organs, such as the heart, so that effective replacement valves can be designed. More generally, simulation is increasingly being used in the design of patient-specific treatments.

- In the **entertainment** industry, the simulation becomes the actual product. This sector is expected to grow rapidly over the next 10 years. The technology currently used in computer games to generate a high-definition, audio-visual consumer experience that is also highly interactive will be used to deliver a wider range of content to a much larger customer base.

- In the **telecommunications** sector, accurate simulation of the propagation of microwave, radio and optical signals, through cluttered environments and various weather conditions, is used to improve the management of the radio spectrum.

- In the **paper** industry, simulation is used to understand the flow of water and wood pulp through a papermaking machine. In particular, the dilute paper stock flows rapidly through a filter, a flow box, a press and finally a drying cylinder. Understanding the nature of these flows can help to improve the design of the process.

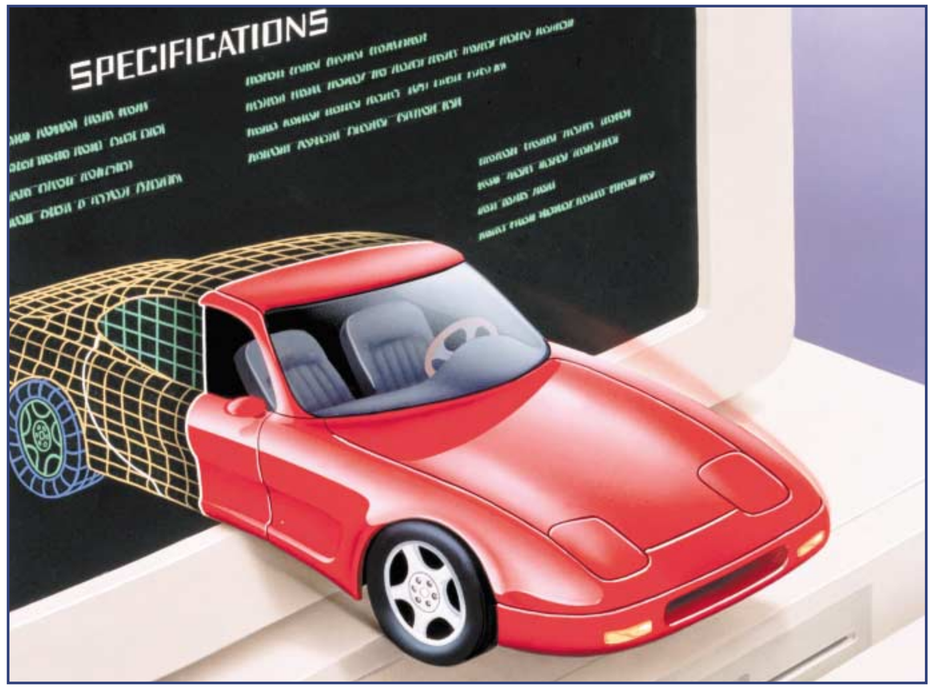
While the various sectors of UK industry have different expectations of simulation, their main needs, and associated challenges for the mathematical sciences may be summarized as follows.

- There is a continuing need for *greater physical realism*. Many simulations lack sufficient realism because good models do not exist, or because model data are uncertain or incomplete, or because the problem is computationally intractable. How can approximate solutions be found that have sufficient accuracy?

- Improving the realism of a simulation usually entails an associated increase in the length of time required to carry out the simulation. There is a need for a *rapid response* to changes in the model parameters. The need for efficiency is particularly important in the entertainment industry, where the constraint that simulations must be carried out in real time and on commodity hardware must not affect the visual experience associated with a computer game. How can engineering designs be taken as input to accurate physical simulations, automatically and efficiently?

- Expensive models on fine scales often need to be replaced with effective coarse-scale descriptions when computation on the fine scale is not feasible. This suggests *hierarchical models*, in which a small, fine-scale simulation can be used to verify a much larger, coarse-scale model. What are the systematic approaches for treating multiscale phenomena?

2.2 Multidisciplinary design



Multidisciplinary design involves bringing together a collection of design tasks that need expertise from several disciplines, in situations where the challenges presented by their integration are comparable to the challenges in each of the tasks separately. The parts of the UK economy involved in large, multidisciplinary projects include:

- The **aerospace** industry, where the development of a new aircraft requires fluid mechanics to model the lift on its wings, elasticity theory to understand its structural integrity, electromagnetic theory to assess its susceptibility to electromagnetic interference, and electronics and software engineering for instrumentation and control. Similarly, the design of modern jet engines requires fluid mechanics, combustion chemistry and materials science.

- The **automotive** sector, where the main motivations in engine design are to improve economy and to decrease emissions; computational fluid dynamics is used to understand the reactive flow through the combustion chamber. Aerodynamics and crash performance determine the shape of the car. The choice of materials used in the manufacture of the car takes into account criteria for crash performance and acceptable levels of vibration and noise. As an example of the competing requirements, good handling demands little or no fore-and-aft compliance in the suspension, but there will be excessive noise and vibration unless there is some compliance.

- The **construction** industry, where the design of a new building involves meeting regulatory requirements for structure, safety and environmental impact, addressing issues of comfort such as adequate ventilation and heating and also taking into account the functional requirements of those who will use the building.

- The **medical** sector, where robotics is an increasingly important area. Robots able to carry out either routine or remote surgery are complicated to design, needing skills from a number of disciplines.

- The **pharmaceuticals** sector, where the design of drug delivery systems requires an understanding of both the mechanical properties of a device and the way in which the drug is delivered in the form of an aerosol or a powder.

The main challenges in multidisciplinary design are:

- Supply chain management and communication, to ensure that decisions taken by each task are communicated to the rest of the project. In multidisciplinary design projects, the individual tasks are often each constructed with a clear goal, requiring only one or two areas of expertise. How can the outputs from different tasks be reconciled?

- Multidisciplinary optimization, to ensure that an optimum design can be selected, given a large number of possibly competing requirements. The traditional approach is to circulate a design between the different tasks, and to hope that a convergent design is obtained. Within each task, optimization often depends on a large number of design variables, leading to expensive and time-consuming computations. How does one develop systematic approaches to facilitate design choice?

- Robust engineering design, where the main concern is the ability to withstand potential uncertainties in the manufacturing process or changes in the operating environment. How should products and processes be designed so that they are robust against variations in production and in their use?

2.3 Uncertainty and risk

Uncertainty, in this context, is the lack of sufficient information to allow a system or process to be well understood, even in principle. Risk is the potential for the realization of unwanted, adverse consequences to human life, health, property, profits, or the environment, and is often estimated using probabilities to measure the likelihood of some hazardous event. They are clearly related concepts. Forecasting in the presence of uncertainty and the estimation of risk are issues in most sectors.

- In the **manufacturing** industries, uncertainty arises from process variability and impacts on quality control. This problem is particularly acute in sectors where the number of units produced is relatively small and there is a high reliance on skilled human labour. In these cases, variability is inevitable and often hard to quantify. Small variations in production can have a large impact on product performance. It is also an issue in the paper industry, where uncertainty arises primarily from variability in the raw material (i.e. wood pulp).

- In the **banking and finance** sector, uncertainties in financial markets, interest rates, foreign exchange rates and the prices of basic commodities provide business opportunities. An important role is played by financial derivatives based on some underlying asset. Determining a fair price for complex derivatives, depending on a number of underlying assets, and quantifying the associated risks are challenging areas for finance, where computer simulation is often used.

- In the **insurance** industry, large-scale hazards are of critical importance in determining profitability. The quantitative assessment of insurance risk depends on accurate modelling of unusual or extreme events, which could be extreme weather conditions, other natural disasters, terrorist attack, company collapse, etc. The so-called tails of the underlying probability distributions are then of greatest interest. Similar issues arise in **finance**: for example, when the foundations for modern portfolio theory were laid in the 1950s, it was assumed that returns follow the so-called normal distribution. Although this is a reasonable assumption, it is now well known that the tails show significant deviations from normality and this has implications for the risk involved.

- In the **food** and **agriculture** industries, understanding and managing risk is particularly important. Recent, high-profile examples of risks to animal health and food safety are the foot-and-mouth outbreak and BSE. Weather-related risk to crops is a further, ever-present source of uncertainty in agriculture.

- In the **retail and distribution** sector, forecasting is crucial. Too much stock leads to wastage and too little to customer dissatisfaction, both of which have a direct impact on profitability. The demand for some products, such as ice cream, depends strongly on the weather and consequently there is a growing market in weather derivatives. The ability to forecast the turnover of a potential new store and how it depends on location is important in planning for expansion. The **transport** sector faces similar challenges, in planning for uncertain future capacity. Too little capacity adversely impacts performance and safety, and too much is a wasted resource.

- In the **electricity** industry, planning and operations have always been faced with uncertainty. However, deregulation of the industry and the creation of new markets has introduced substantial new uncertainties on both short and long timescales. Regulatory pressure and increased competition will put an even greater emphasis on the management of uncertainty and risk. Accurate forecasting of capacity in the **telecommunications** sector is important for very similar reasons.

- In the **oil and gas** industry, there is a need to forecast the reserves of a potential new oil or gas field from limited data, in order to decide whether its exploitation is economically feasible. The same considerations apply to the **mining** sector in deciding where to develop a new mine. In both these industries, large up-front investment is required before the natural resource can be extracted, and so errors in the estimates of the reserves can be very expensive.

- In many industries, including **transport**, **nuclear**, **construction** and **chemical**, safety is of paramount importance and so there is considerable emphasis on assessing, quantifying and reducing the associated risks. Companies must take safety into account when making investment decisions.

One of the great achievements of mathematics is the development of tools that allow risk and uncertainty to be quantified and controlled. These tools are based on models of randomness, known as stochastic models. Their further development and their application to a wider range of problems and industrial sectors will result in increased performance and profitability. Dealing with uncertainty often requires a combination of stochastic methods and simulation methods. Key challenges include the following.

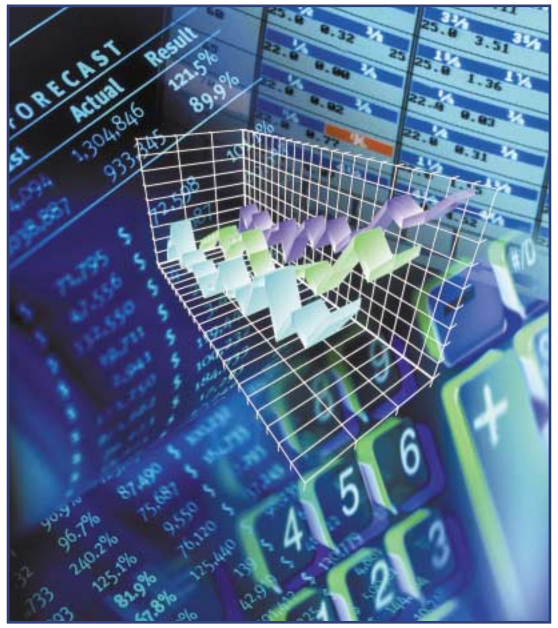
- Even after choosing an appropriate model, there is the challenge of accurately `estimating parameters` of interest from the available data, which may be biased, noisy and incomplete. For example, in the oil industry, permeability is estimated from well logs and seismic surveys and then used in simulation tools by reservoir engineers. What are the robust and reliable methods for estimating parameters in increasingly complex models?

- In many situations, it is important to understand the sensitivity of systems to uncertainties. Although it may be very difficult to model the uncertainty, it may be possible to improve the design so that there is lower sensitivity to it. The electromagnetic compatibility of a car or aircraft is a good example. What techniques can be developed for estimating and minimizing the impact of uncertainty?

- Perhaps the greatest source of uncertainty is human behaviour. For example, in the retail sector, the design and marketing of new products are heavily influenced by human behaviour, as are the 'in-house' issues of manufacture and quality. Some companies are beginning to use quantitative techniques to model human behaviour in the context of product design. How does one develop appropriate, quantitative models of human behaviour?

2.4 Data

Breakthroughs in sensor technology, coupled with advances in data storage, are leading to the generation of unprecedented volumes of data. The challenges to industry are 'How can this data be managed, what can be done with it, and how can it be done more quickly?'



- In the **banking**, **insurance and finance** sector, data is collected both on customers and on the financial markets. In insurance, the existence of extensive data on an individual applicant could lead to personalized insurance policies.

- In the **retail and distribution** sector, data is collected through loyalty card schemes to understand the needs of customers, and thereby focus business and minimize costs.

- In the **oil and gas** industry, companies manage oil reserves by carrying out extensive geological characterization of their reservoirs, and also collect daily data from monitoring boreholes and production wells.

- In the **pharmaceuticals** sector, companies must demonstrate the safety of new drugs through extensive trials, which generate large amounts of data. Data quality can vary from highly accurate to noisy, and may even be non-numerical. Other aspects of research that involve extremely large quantities of data are biological databases and genome bioinformatics.

- In the **telecommunications** sector, data is collected on customer behaviour and there is potential access to further data, such as the location of mobile telephones. For example, in Germany information on mobile telephone locations is being used in a trial traffic management scheme.

- In the **transport** sector, train-operating companies collect information on customers and on the state of the infrastructure as a routine part of operations. Large amounts of data concerning the positions and movements of aircraft are gathered and analysed in the air traffic industry for research into capacity, safety and technology development.

- In the **manufacturing** industries, it is increasingly common to collect data from the production line as part of routine operations, although it is often not exploited systematically.

- Although not an industry in the usual sense, **crime detection** often depends on the ability to identify similarities between different data sets. Handling large volumes of data from a variety of disparate sources poses substantial problems. For example, determining whether illegal material is being stored on a personal computer currently involves a manual search through entire hard drives. Moreover, digital closed-circuit television will in future create large amounts of data that will be used in the detection of crime and that can be analysed with the help of mathematics.

- Finally, **public services**, such as the National Health Service and the Police, are judged by measures that attempt to quantify their performance. The data that is input to those performance measures is often imprecise and may be subject to bias. For example, how is one to quantify patient recovery, the effects of homelessness or the number of crimes prevented by having a policeman on the beat?

In summary, huge data sets, often running into terabytes, are now collected routinely. A large part of the information technology industry is concerned with their efficient storage and retrieval. Mathematical challenges include the following.

- Algorithmic advances, coupled with the continued growth in the power of computers, offer new possibilities for visualising, analysing and otherwise 'mining' data. How can these techniques be developed and used to extract information of commercial value?

- Limitations in sensing capability can adversely affect data quality, and present mathematical challenges for the analysis and the use of such data in associated mathematical models. How should mathematical models take account of data quality?

- Data is commercially valuable. Many companies already deliver their products and services in the form of digital data sets (e.g. films on DVD and music on CD) and even more will do so in the future. This raises important issues of security, privacy, ownership and access, which are essentially mathematical in nature, and which will have a profound impact on the future of business and industry, and indeed on the lives of individuals. Where are the tools to enable business to function efficiently and securely in a data-rich economy?

2.5 Complex systems

A complex system is made up of a large number of constituent components. The essential feature is that the system's sheer size and the interactions between components give rise to an overall behaviour that cannot easily be understood in terms of the behaviour of the individual components. In some of the following examples, the behaviour of the individual components is itself poorly understood, notably when there is a human aspect.

- In the **aerospace** industry, a modern passenger aircraft has millions of components that constitute the various systems. The design must ensure that these systems work together as intended and that there are no dangerous interactions that might compromise performance or safety.
- In the **finance** industry, international markets are complex systems consisting of many buyers and sellers and their interactions generate endogenous (internal) risk quite apart from exogenous risk.
- In the **food and drink** sector, many products are complex mixtures that must be carefully designed to produce the desired texture, consistency and flavour.
- In the **chemical** industry, many personal care products such as toothpaste, skin cream, hair care products and soaps are complex formulations made up of components selected from a wide range of possibilities.
- In the **retail and distribution** sector, the customer base is a complex system consisting of many individual consumers. Identifying and understanding the buying patterns of groups of consumers helps retailers to improve the service that they offer.
- In the **electronics** industry, both hardware, which involves very complicated circuits, and software are becoming more complicated. The increasing complexity makes new systems difficult to design, although the industry has been remarkably successful in overcoming these difficulties.
- In the **pharmaceuticals** and **medical** sectors, the human body can be regarded as an extremely complex system in which it is difficult to predict the effects of new drugs. A greater understanding of the way in which complex systems behave may be extremely valuable to these sectors in the future.
- In the **information technology** sector, software applications often contain many millions of lines of code. This complexity often makes it difficult to detect programming errors, especially ones that depend on the simultaneous occurrence of relatively rare conditions.
- In the **telecommunications** sector, networks support millions of devices. Poor response times caused by heavy network loading can be due to undesirable emergent behaviour that is similar to traffic congestion on roads.
- In the **transport** sector, the road, rail and air traffic control systems are examples of complex systems. Some forms of traffic congestion can be caused by undesirable emergent behaviour.

In compiling the roadmap, one leading figure in the information technology field emphasized that 'complexity is the single biggest challenge we face in taking technology forward'. There is an urgent need to understand how complex systems behave and to move towards the design of systems in which the required global behaviour emerges without the need for centralized command and control.

- A promising approach to dealing with the problems of complexity in computer systems is to model them on self-regulating biological systems. This approach is sometimes called autonomic computing because of its similarities with the human autonomic nervous system. What are the essential features of these biological systems? How can they be used in the design of large computer systems and networks?
- Failure in complex systems can be due to the simultaneous occurrence of a number of rare events. Identifying these problems at the design stage and building in system reliability is very difficult, owing to the sheer complexity of the complete system. What methods and tools can designers use to understand and eliminate failures of this kind?
- Small disturbances to a few components in a complex system can propagate through the system with dramatic, even catastrophic results. Stock markets crashes may be an example of this. How can such events be anticipated and controlled?

2.6 Market behaviour

Companies operate and compete in many different types of market. Telecommunications companies, energy distributors and train operating companies have regulated markets, where constraints are placed on their business operations. In contrast, retailers generally operate in a more open market. In all cases, companies must understand the markets in which they operate.

- In the **aerospace and defence** sector, there is increasing pressure to take a forward view about procurement strategy. In order to succeed in such an environment, companies must understand better the markets in which they operate.

- In the **banking**, **insurance and finance** sector, models of financial markets help to manage risk and to price financial products. Sophisticated mathematical models of financial market dynamics have been developed, and are used extensively for trading, risk management and regulation.

- In the **food and drink** sector, the supply and prices of agricultural commodities, along with their associated markets, have dramatic impact on the overall cost of production.

- In the **retail and distribution** sector, understanding of the market is crucial in determining business strategy.

- In the **energy and utilities** sector, the electricity generators and end-users operate in an extremely fluid market, where contracts for the sale and purchase of electricity are exchanged on timescales from months to half-hours. The electricity system must accommodate these fluctuations in the market, as well as the longer-term energy and environmental influences on the market.

- The **pharmaceuticals** sector needs to be aware of the value of new systems in terms of patient benefit. This understanding is developed using data analysis, market modelling and user feedback.

- In the **telecommunications** sector, there are auctions for licences that give rights over portions of the radio spectrum. Similarly, in the **transport** sector, operating companies must compete for franchises. Accurate estimates of future revenues are needed to assess the value of such rights.

The challenges for mathematics are to help elucidate market behaviour and to guide the design of new markets.

- The Internet is leading to a revolution in the marketplace. There is a move towards automated markets, where an algorithm enables a company to put out an invitation to tender and then select a supplier. A mathematical model is used to choose the best supplier. How do these automated markets behave? How can reliable algorithms for the market be developed?

- Many markets are complex systems in the sense of section 2.5, and so the possible effects of regulation are of great interest. For example, how can a regulator help to avoid undesirable emergent behaviour? How can a regulator balance the interests of suppliers and consumers? When is it advantageous for a regulator to coordinate transactions in a market?

- The role of uncertainty in markets is also important. How do market participants respond to uncertainty? How is the outcome of the market dependent on the quality of the data that is available?

Table 1: The links between economic sectors and scientific themes

	Simulation	Multi-disciplinary design	Uncertainty and risk	Data	Complex systems	Market behaviour
Aerospace & defence	✔	✔	✔	✔	✔	✔
Automotive	✔	✔	✔		✔	
Banking, insurance and finance			✔	✔	✔	✔
Food and drink	✔	✔	✔	✔	✔	
Chemical	✔		✔	✔	✔	
Construction	✔	✔	✔		✔	
Retail and distribution			✔	✔	✔	✔
Energy and utilities	✔		✔	✔	✔	✔
Electronics and computer hardware	✔				✔	
Pharmaceuticals	✔	✔	✔	✔	✔	✔
Medicine	✔	✔	✔	✔	✔	✔
Textiles	✔					
IT and software				✔	✔	
Leisure and entertainment	✔		✔	✔		
Mining	✔		✔	✔		
Telecommunications	✔	✔	✔	✔	✔	✔
Transport	✔	✔	✔	✔	✔	✔
Paper and forestry	✔		✔			
Crime detection			✔	✔		

Case study: Fluid flow in milking equipment (page 25)
Case study: Radio spectrum congestion (page 26)

3 Mathematical and computational methodologies

Having identified the relevant themes, it can now be examined how the associated industrial challenges fit into the research landscape. The six themes of the previous section may be addressed using various combinations of research methods. By linking the two together, the roadmap can be completed. The broad groupings of research methods that are presented here have been devised in extensive consultation with industrialists and academics. They are called 'methodologies' to indicate that they each encompass a variety of specialist tools and methods. In contrast to the approach of taking industrial sectors from stock market classifications, there is no standard classification of research methods that is suited to the purposes of the roadmap.

The linkage between themes and methodologies is given in table 2 on page 24. Against a constantly evolving industrial background, the list of methodologies cannot anticipate every technique that might be useful, but it can highlight the broad groupings that are most proven or promising in terms of economic impact. Note also that 'computing' in the context of the roadmap means the use of computers to implement mathematical ideas in ways that extract their economic value; it does not, for example, extend to the discipline of software engineering in its own right or to the design of computer architecture.

3.1 The importance of modelling

When applying mathematics and computing to an industrial problem, it is vital to agree upon a mathematical 'model' of the problem, to which mathematical and computational techniques can then be applied. The model is the mathematical framework in which the problem will be tackled. Its design must identify the most important issues in the problem, take into account the amount of computation that is needed in the solution, and bear in mind the availability of the industrial data that is needed to use the model. The construction of the model is a process in its own right and so the roadmap makes a distinction between **modelling methodologies**, used to construct models, and **analytical methodologies**, used to analyse models. The two are related in the following three-phase process, which is the general pattern for applying mathematics and computing to industry:

(a) The initial posing of the question by industry and the early dialogue with mathematical scientists, leading to the design of an appropriate mathematical framework, called a model, in which to work on the problem. This phase requires **modelling methodologies**.

(b) The theoretical analysis of the chosen model, drawing upon the necessary industrial data associated with the problem at hand to make detailed predictions. This phase requires **analytical methodologies**.

(c) Further dialogue between industry and the science base, to validate and, if necessary, update the model and to act upon the theoretical predictions of phase (b), again employing **modelling methodologies**. Model validation is especially important when there are significant uncertainties in the model or in the measured data.

When the process is viewed in this way, the importance of modelling is apparent. The quality of the model, in terms of its accuracy, ease of use and range of use, is crucial. Good mathematical analysis done on a poor model is unlikely to be of much value.

3.2 Modelling methodologies

Modelling methodologies are used to construct mathematical models. The six roadmap themes raise different modelling issues, requiring different methodologies. The most well-established theme is that of simulation (see figure 1), in which the relevant methodologies are generally based on the laws of physics and chemistry that govern the underlying industrial processes. The main concerns here are the timescales and length scales of the process, together with the industrial operating conditions. The latter might involve, for example, the temperature at which the process is carried out. Operating conditions appear in mathematical models as different 'parameter regimes'.

In contrast, for the other five themes there may be no natural basis on which to build a model and perhaps not even a clear question in the first place. Modelling methodologies will be more diverse, with a multidisciplinary dialogue required to identify the most appropriate mathematical framework. Some of the relevant methodologies are listed below, but there are lacunae that call for vivid imagination to discern the new theoretical structures that are needed. For example, in industrial settings that call for a high level of judgment, models that are used to support decision making should be able to take account of these subjective judgments, especially when operational decisions must be taken quickly.

Modelling of scale and parameter regimes in mechanics: in all modelling of physical systems, judicious choice of timescales and length scales is vital. This demands knowledge of behaviour at many scales and especially whether the behaviour is turbulent (chaotic) or laminar (predictable). Examples abound in the theme of **simulation** and these arise also in **multidisciplinary design**, **uncertainty and risk** and **complex systems**.

Modelling of scale and parameter regimes in electromagnetics/optics: here there is an even greater need to understand scale variations. As well as the relative scales of wavelength and structure size that are so important for **simulation** in the aerospace, automotive and telecommunications sectors, there is the additional requirement of deciding when and how quantum effects need to be modelled, for example in semiconductors and superconductors.

Modelling of scale and parameter regimes in heat/mass transfer and chemistry: this methodology is concerned with **simulation** to make accurate predictions in applications as diverse as baking of bread, alloy microstructure, fire safety and atmospheric contamination. There is increasing use of fine-scale (atomistic) computational models, often complemented by more tractable mathematical models.

Modelling of scale and parameter regimes in biology and medicine: in applications of molecular biology, **simulation** using large computational models is dominant; elsewhere,

there is demand for large-scale modelling of drug-delivery systems, tissue engineering, implants, medical imaging and surgical strategies. The understanding of multi-scale effects, from subcellular to whole body, is important for the optimization of treatment regimes; the interactions that occur between disease, treatment and immune responses lead to highly **complex systems**.

Market and financial modelling: spectacular practical advances are possible if realistic models are available for **market behaviour**, especially the human aspects. These models require first-hand knowledge of the causes of **uncertainty and risk**, dealer behaviour and the appropriate use of utility functions. In addition to modelling markets, they are used for assessing credit risk, and valuing projects and businesses through so-called 'real options'.

Discrete modelling: in **complex systems**, discrete models describe the interactions between individual components, such as timing constraints in process scheduling, the separation between vehicles on a motorway and the potential interference between radio channels. In the theme of **market behaviour**, they describe the interactions between potential trading partners.

Stochastic modelling: the above methodologies frequently need to be adapted to cater for randomness in the model parameters and associated data. In such cases, it is crucial to identify which stochastic inputs are most important in determining variability in performance. Stochastic modelling underpins the study of **uncertainty and risk** and also many problems in the theme of **data**.

3.3 Analytical methodologies

Analytical methodologies are applied to mathematical models, in order to make detailed predictions of industrial value. They are where the specialist mathematical and computational expertise of the science base comes to the fore. When the relevant models can be drawn from past experience with similar problems, it is likely that the appropriate analytical methodologies are also well established. Industry's needs can then be met by transferring existing knowledge to the new domain of application. On the other hand, when the models themselves are novel, there is often an associated need to create new analytical tools with which to tackle them.

In what follows, analytical methodologies are further divided into those concerned with the **analysis of models** and those concerned with the **analysis of data**, reflecting the distinction between models and data that was made in section 3.1. This division is not a crucial element in the roadmap, but it can serve as confirmation that an appropriate mathematical focus is being used; in particular, statistical methods feature prominently in the analysis of data.

Analysis of Models

Numerical differential equations: this methodology concerns the solution of lumped (ordinary differential equations) or distributed (partial differential equations) parameter models in numerical or graphical form, with associated error estimates. It is vital for making predictions from all but the simplest **simulation** models. There are often associated issues of computational geometry, meaning the efficient and realistic representation in computer software of complex geometric shapes. In addition, sensitivity analysis delivers quantities that enable formal statistical validation and testing to be carried out.

Asymptotics: the disparate sizes of parameters in models may be exploited to obtain concise and explicit formulae for making predictions in **simulation**. These techniques provide quality control on numerical prediction from a wide range of models, and include the increasingly important area of homogenization, in which models with smoothly varying parameters are derived from ones with rapidly varying parameters.

Stochastic analysis: statistical analysis and the analysis of stochastic differential equations are both vital for validating lumped or distributed parameter models with random data and for deriving efficient computational schemes for such models. Stochastic differential equations are at the heart of almost all modern financial models, and the interplay between statistics and stochastic analysis is at the heart of the analysis of **uncertainty and risk**.

Optimization: a generic requirement in industry is to maximize performance, reliability and robustness by finding appropriate design parameters, taking into account the industrial design constraints. Optimization is often used to analyse **simulation** models, but there are variants suited more to other themes. For example, multiobjective optimization is especially relevant when analysing models of **multidisciplinary design**, and optimal control is used to provide real-time control in response to current operating conditions.

Combinatorial optimization: discrete models, and especially models of **complex systems**, present particular challenges when used to identify optimal strategies. The issue of 'computational complexity' is crucial when there are associated challenges in **data**. The practical computational goal is often a good approximate strategy rather than an optimal one, and modern techniques such as simulated annealing and genetic algorithms are valuable here.

Stochastic simulation: large computer simulations can be used to make predictions in **complex systems** containing **uncertainty and risk**. Stochastic simulation is heavily used in the assessment of financial risk, for example by credit agencies to value projects, assets and businesses. It is used more generally to study **market behaviour**. A key challenge is how to extract information from the simulation without excessive computation.

Game theory: this methodology is aimed specifically at determining the best policies in strategic interaction and at predicting the outcome of strategic decision-making, especially when human behaviour is an important component. It lies on the interface of mathematics and economics and has a key role to play in the themes of **complex systems** and **market behaviour**.

Algorithmics: algorithms are the basic mathematical building blocks from which computing systems are constructed. The design, analysis and implementation of new algorithms are essential for innovation of computing infrastructure, tools and applications. The mathematical analysis of algorithms impacts on all the roadmap themes.

Analysis of data

Inverse problems and parameter identification: it is a generic problem to identify the parameters of a model from data and to estimate the error bounds or confidence intervals for them. This methodology is applicable across all six scientific themes, because most mathematical models have parameters that must be determined.

Pattern recognition: it is often possible to make decisions based on perceived similarities of the available data to previously encountered examples. The uses of pattern recognition include data compression, data retrieval, computer vision, medical imaging, remote sensing and even deciding whether or not to give credit to a shopper. It is one of the key methodologies for exploiting **data**, where it has been heavily influenced by neural networks. It also has important roles to play in **complex systems** and **market behaviour**.

Signal processing: a signal is information transmitted using acoustic or electromagnetic waves, and signal processing is the extraction, coding, compression and protection of such information. Most signal processing is carried out using digital technology and involves a wide range of mathematical techniques. It underpins the modern communications industry and the theme of **data**, but is applicable to other areas, including the detection of fault 'signatures' in **complex systems** such as aircraft engines.

Data mining: large **data** sets require careful investigation in order to discover and explain patterns, special features and causal connections. Data mining contains many ad hoc methods, but the better ones are underpinned by statistical theory. A particular challenge is to apply data mining techniques to time-dependent data and so understand the evolution of **complex systems**.

Time series analysis: streams of data gathered over time may be analysed to determine the evolving nature of the phenomena that they represent, including the prediction of future values. Time series analysis has many applications, such as economic and market trends (**market behaviour**), weather and climate patterns, control mechanisms and all types of forecasting (**uncertainty and risk**).

Cryptography: this methodology is concerned with disguising messages or information so that only certain people can see through the disguise. The closely related area of cryptanalysis seeks to understand the disguised messages. Cryptography is a vital technology in the modern commercial world where **data** and information need to be protected from unauthorized access or use.

Table 2: The links between scientific themes and mathematical methodologies

Modelling Methodologies	Simulation	Multi-disciplinary design	Uncertainty and risk	Data	Complex systems	Market behaviour
Scale and parameter regimes in mechanics	✔ (yellow)	✔	✔		✔	
Scale and parameter regimes in electro-magnetics/optics	✔	✔				
Scale and parameter regimes in heat/mass transfer and chemistry	✔	✔	✔		✔	
Scale and parameter regimes in biology and medicine	✔	✔	✔		✔	
Market and financial modelling			✔	✔		✔ (green)
Discrete modelling		✔			✔ (green)	✔
Stochastic modelling	✔		✔	✔		✔
Analytical Methodologies						
Analysis of Models						
Numerical differential eqs.	✔ (yellow)					
Asymptotics	✔ (yellow)					
Stochastic analysis	✔		✔		✔	
Optimization	✔ (yellow)	✔			✔	✔
Combinatorial optimization				✔	✔ (green)	✔
Stochastic simulation	✔		✔		✔	✔ (green)
Game theory					✔	✔ (green)
Algorithmics	✔	✔	✔	✔	✔ (green)	✔
Analysis of Data						
Inverse problems and parameter identification	✔	✔	✔	✔	✔	✔
Pattern recognition			✔	✔	✔	✔
Signal processing			✔	✔	✔	
Data mining				✔	✔	✔
Time series analysis			✔	✔		✔
Cryptography			✔	✔		

(yellow) Case study: Fluid flow in milking equipment (page 25)
(green) Case study: Radio spectrum congestion (page 26)

Case study: Fluid flow in milking equipment

Ambic Equipment Ltd is based in Witney, Oxfordshire, and is a typical SME (Small or Medium-sized Enterprise), with 33 employees and an annual turnover of around £2.5M. It manufactures equipment for use on dairy farms, mostly for the detection and prevention of mastitis, an infection of the udder, which is the largest preventable cost to the dairy farmer. Over 80% of the company's production is exported, and Ambic has an excellent record for innovation, winning a Queen's Award for Technology in the 1980s.

Recently, Ambic recognized that there were unsatisfied customer and market requirements, necessitating additional research and development, which the company could not undertake alone. The key question concerned the flow of milk in the equipment used in milking parlours. This flow is unsteady, because it is driven by the repeated pulses of the milking machine, and is also a 'two-phase' flow, consisting of both milk and air. These features lead to difficulties in controlling the dynamics of the flow.

Ambic met initially with a small group of mathematicians from the Smith Institute and OCIAM (the Oxford Centre for Industrial and Applied Mathematics), including some with expertise in two-phase flow in the oil industry, where the flow of mixtures of oil and gas must be understood and controlled. The transfer of ideas from oil and gas flows to milk and air flows proved to be strikingly informative in selecting appropriate mathematical models that can be used to simulate the flow.

A follow-up meeting was held as part of the Faraday-sponsored European Mathematics-in-Industry Study Group in 2001 at Keele University, where mathematicians from several universities considered the problem further and also studied the flow on a video supplied by Ambic. Several new ideas for the control of the flow immediately emerged. The most promising idea was pursued by Ambic, tested in a simple experiment, and was found to achieve precisely what was needed.

Ambic is now developing this concept, with continuing advice and input from OCIAM and the Smith Institute. The project is being supported by a DTI Smart award and is expected to lead to significant improvements in the ability of dairy farmers to monitor the health of their herds. This in turn is expected to be reflected in significant increases in Ambic's turnover and profit, as well as employment and exports. The speed of feedback and smooth transitions from one phase of research and development to the next have allowed the project to progress quickly and gain a competitive advantage.

Case study: Radio spectrum congestion

Radio technologies are among the key drivers of the ongoing communications revolution, which is transforming business and society in a way that is comparable to the industrial revolution in the 19th Century. Radio offers unparalleled flexibility for information distribution and access, and is the only means of satisfying the growing demand for mobile services. Cutting-edge commercial applications include 3G mobile communications, broadband internet access and digital broadcasting. Beyond commercial services, radio is also crucial to the effective operation of the armed forces, air traffic control, emergency services and the police.

The central challenge to be addressed is that all these applications rely on the same, limited natural resource, namely the electromagnetic spectrum. It has been recently calculated that the value to the economy of the radio spectrum is in excess of £20 billion annually[1] and so even small percentage improvements in either the spectrum requirements for new services or the spectrum efficiency of existing ones will have heavy economic impact.

Radio communications have been established for over 100 years, and for all but the last few years spectrum has been made available on a first-come first-served basis. The main focus has been on ensuring that radio users can operate without interference from other users or from the environment. Improved engineering of radio hardware has reduced susceptibility to interference, helped to control unwanted radio emissions, and opened up new portions of the spectrum; but these efforts have been outstripped by the demand for new services. The spectrum is now in a state of acute congestion and new tools are being introduced to ensure its effective management for best economic advantage.[2]

Many countries, including the UK through the Radiocommunications Agency, are developing innovative approaches to spectrum management, involving a mix of regulation, pricing and market forces. Where appropriate, spectrum auctions and the introduction of a secondary market in the form of spectrum trading are giving users genuine economic incentives to use spectrum efficiently.

The UK is one of the world leaders in developing the associated mathematical underpinnings, which include analogues of the famous 'four-colour' problem and advances in mathematical economics. The Smith Institute is at the forefront of this activity. Spectrum management is now less of a problem in engineering and more of a problem in simulation, complex systems and market behaviour. Mathematics provides the foundations on which this dramatic change in emphasis is being implemented, to the benefit of all radio users.

[1] *The Economic Impact of Radio*, Radiocommunications Agency, February 2001, www.radio.gov.uk
[2] *Review of Radio Spectrum Management*, Martin Cave, March 2002, www.spectrumreview.radio.gov.uk

4 Establishing successful collaborations

It now remains to address the question of what steps are needed in practice to give industry the maximum benefit from interaction with the mathematical sciences. How does the roadmap help to identify the most promising collaborations? How do organizations such as the Smith Institute help in establishing and maintaining collaborations? What are the features of successful collaborations and what will they look like in the future?

Using the roadmap

Mathematics is used in industry to address both long-term 'challenges', which are generic issues of strategic importance, and short-term 'problems', which are focussed operational issues. For example, the catalogue on pages 3 and 4 mostly consists of problems, while major challenges are given by the concluding questions in the presentation of each roadmap theme in section 2. Innovative mathematical solutions of problems and challenges are equally important in terms of economic impact, and the roadmap addresses both, by guiding the matching of industrial requirements with mathematical and computational methodologies, using the six roadmap themes as an intermediate point.

Although the roadmap is applicable to both problems and challenges, its greater value is likely to be in the context of challenges, where the choices of best mathematical models are often far from clear at first sight. In contrast, problems tend to have a much clearer specification. They have traditionally been the focus for most industrial-academic interaction, and there is often sufficient background knowledge on both sides of the collaboration that one can immediately identify the best methodologies without formally going via the themes. In figure 2, this distinction is shown by lighter connecting arrows.

The connections between industrial requirements and methodologies are made by using tables 1 and 2 in conjunction. Understanding the industrial context will lead to identification of the relevant themes, and here table 1 serves as a guide, by indicating the themes that have been identified by each industrial sector in the roadmap interviews. Referring then to table 2 and section 3, one can determine the broad methodologies that are most likely to be of value.

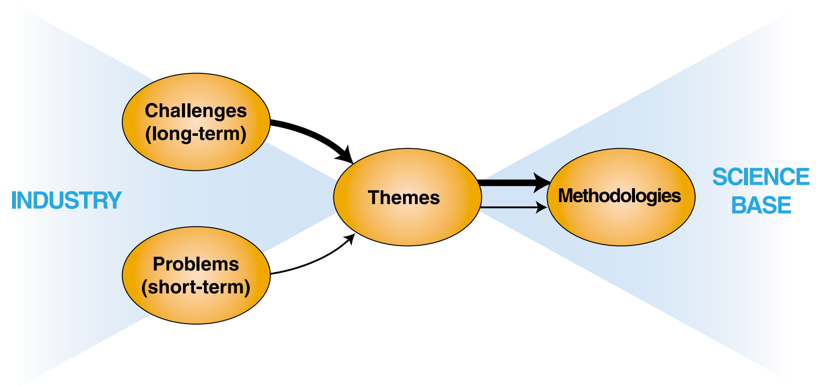


Figure 2: Using the roadmap

It should be recognized that the roadmap is an **industrial roadmap for mathematics, not a mathematical roadmap for industry**. The description of methodologies in section 3 does not therefore contain a detailed scientific survey of all industrially relevant mathematics available in the UK, and neither would such a survey be appropriate in this document. The best specific techniques within the broad roadmap methodologies can only be identified case-by-case through careful dialogue. In this way, it will become clear where existing techniques must be extended or totally new approaches developed. Since table 2 suggests that relatively few methodologies exist to address the themes of multidisciplinary design and market behaviour, these are particularly promising areas for new activity.

Assistance in establishing and maintaining collaborations

Intermediate organizations such as the Smith Institute offer great flexibility, especially when a wide picture of the academic landscape is needed, or assistance is needed to assemble a collaboration of several partners. The Smith Institute is the leading intermediate organization in the UK for connecting industry and academia in the underpinning disciplines of the mathematical sciences and its objective is to spread their benefits widely in the formulation of industrial strategy and in the development of new products and processes. Institute staff all have backgrounds in the industrial application of mathematics, and are able to participate in every aspect of creating, sustaining and managing industrial-academic interactions.

Case studies

The case studies on pages 25 and 26 illustrate the creation and content of a new collaborations. The coloured highlights in tables 1 and 2 show the corresponding linkages. The flexibility of the mathematical community in bringing diverse skills to these problems has been vital to the success of the collaborations.

Such successes are just the tip of the iceberg. Each of the project titles in the catalogue in figure 1 has a similar story behind it. The key elements in ensuring a competitive edge are capturing industrial requirements clearly, assembling diverse mathematical expertise, providing rapid response, and designing a chain of mechanisms to take the project through to successful completion.

Managing expectations

Business and industry on the one hand and academia on the other use very different metrics to evaluate performance. In business, profitability is the main criterion, on timescales that vary between companies and across sectors. In academia, reputation and influence are usually the main motivations, often associated with the desire to create critical mass in new research areas. Nowadays, these differing motivations are often the subject of formal intellectual property agreements, which address the rights of each party over the collaboration's results. Effective collaborations manage to balance the achievement of specific business objectives with the strategic strengthening of the science base.

Additionally, there is generally a mismatch between the timescales on which industry and academia operate. In industry, many important problems must be solved within weeks or months, but at the other end of the scale there are also large, safety-critical projects with durations in excess of 10 years. The science base, at least in mathematics and computing, has tended to make advances over much more uniform timescales, generally in the range 2-4 years, in keeping with typical funding periods. Effective collaborations manage to balance the timescales of industrial demands with the natural academic cycle.

Leveraging existing knowledge

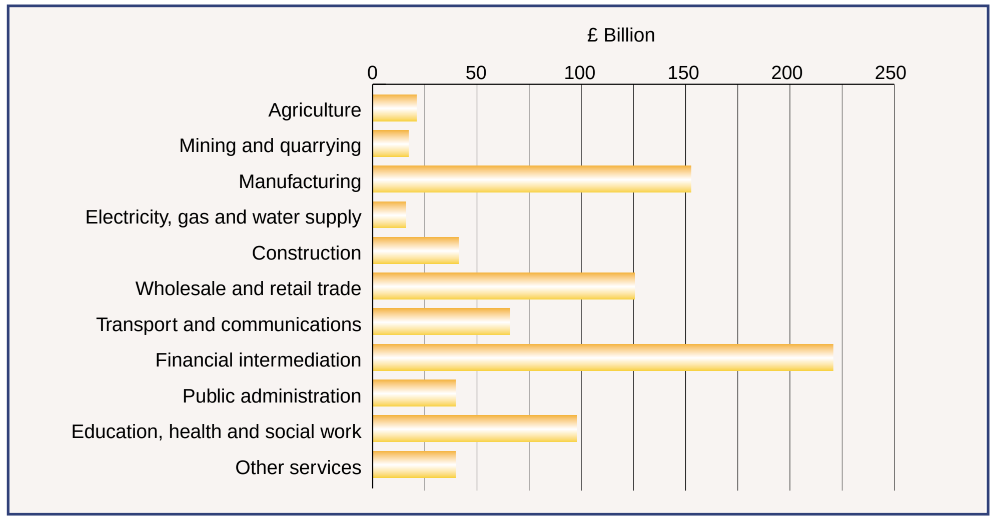
One of the most exciting findings of the roadmap is that advanced mathematics is being deployed in an increasing range of industrial sectors. Retail and distribution is an excellent example where the use of mathematical techniques is more recent than in other sectors, but is now firmly established as being an essential element of commercial competitiveness. The wider range of business applications is coupled with an increase in the number of areas of mathematics that are being called upon, many of which are areas of traditional strength in the science base that have not previously been considered to be of prime industrial relevance. For example, several roadmap interviewees emphasized that techniques from discrete mathematics (graph theory, combinatorics, set theory, logic) are now prominent parts of the industrial mathematician's toolkit, driven by the increasing contribution to the economy of service-sector industries.

A more strategic consequence for the science base stems from the multi-thematic nature of many industrial challenges. Their solution will require innovative combinations of methodologies in long-term collaborative efforts, which industrialists, policy makers and academics can all help to bring about. In deriving the best competitive advantage for the economy, it is clear that closer links will be forged between different branches of mathematics, as they are drawn together to work towards common goals. The roadmap themes do not respect the traditional academic boundaries between applied mathematics, statistics, computing and pure mathematics. **The over-riding conclusion is that in future industry will need flexible access to a more connected academic community**.

Appendix A: Generating the roadmap

In constructing the roadmap, care was taken to obtain views from industrial sectors that are representative of the economy as a whole. The following figure, based on the Blue Book 2001[1], shows the distribution of gross added value by industry for 1999. The three largest sectors are financial intermediation and other business services (£220.6 billion out of a total of £795.0 billion), manufacturing (£152.7 billion) and wholesaling and retailing (£125.6 billion). The importance of the service sector relative to traditional manufacturing is very clear and its relative growth reflects a clear long-term trend in the economy. Over the last 10 years, the average annual increase in the output of the production sector has been 1.1%, against 2.9% in the service sector, and the trend is accelerating. In the light of these figures, particular attention was paid in compiling the roadmap to the increasing importance of the service sector.

Gross added value by sector



The classification of the stock market into business sectors, as used in the Financial Times, provided a further check on the industrial coverage of the exercise, and was also used as a guide when selecting companies to approach for interview. Each interview was preceded by an initial contact with senior figures within the selected companies, during which the purpose of the study was explained. All those contacted were keen to help, and over 95% provided an interview, a few by phone, but the vast majority in person. Several companies arranged for more than one person to contribute in order to give as broad a view as possible.

The six roadmap themes emerged naturally in the first few interviews and the later ones reinforced their relevance across a wide range of different sectors, while giving insight into new applications. Therefore, there is very strong evidence that further industrial consultation at this time would not result in any significant updating of the roadmap.

1 *The Blue Book 2001*, United Kingdom National Accounts, The Stationery Office, London

Appendix B: Further reading

The role of mathematics in science and industry has been the subject of several publications over recent years, although never before in the form of an industrial roadmap. Nevertheless, it is interesting to compare some of the main conclusions.

- In the United States, Wright and Chorin[1] carried out a study of the relationship between mathematics and other sciences. They identify modelling, complexity and size, uncertainty, multiple scales, computation and large data sets as 6 recurring themes. Although their work did not have input from industry, the common threads in the relationships that mathematics has with other academic disciplines bear a remarkable resemblance to the roadmap themes.

- Again in the United States, the Society for Industrial and Applied Mathematics (SIAM) has produced the SIAM Report on Mathematics in Industry[2], which in large part focussed on the role of nonacademic mathematicians. The report is the output from a 3-year study of several hundred mathematicians, scientists, engineers and managers. It confirmed the huge range of application of mathematics, and identified the following characteristics of successful industrial mathematicians: skills in modelling and analysis; flexibility across applications; experience with computing; and the ability to collaborate. From the managers' point of view, the report concluded that the most valued skills are, first, the ability for abstraction and analysis of industrial problems and, second, expertise with the best tools for formulating and solving problems. These findings are echoed strongly in the roadmap.

  A further observation of the SIAM Report is that '*mathematics is alive and well, but living under different names*', meaning that in industry the use of mathematics is often not labelled as such. Similar observations were made by UK industry during the roadmap interviews. Even within academia, cutting-edge mathematics is often done under the guise of computer science, engineering, physics, economics and other sciences.

1 *Mathematics and Science*, Margaret Wright and Alexandre Chorin, National Science Foundation, USA, 1999.
2 The *SIAM Report on Mathematics in Industry*, Society for Industrial and Applied Mathematics, Philadelphia, USA, 1998.

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Dr Andrew Aftelak (Motorola)
Prof Martin Bradley (National Grid)
Dr Peter Brand (GlaxoSmithKline)
Mr Keith Brandon (formerly Tesco)
Dr Chris Brown (BP)
Dr Andrew Burgess (ICI)
Prof Geoff Callow (Motor Industry Research Association)
Ms Bronwyn Curtis (Bloomberg)
Dr Vince Darley (EuroBios)
Dr Graham Dean (QinetiQ)
Mr Alan Gibbon (Mining Industry Research Organization)
Dr Nick Granville (Smith+Nephew)
Prof Peter Grindrod (Numbercraft)
Dr Richard Golding (Institute of Physics)
Dr Dougal Goodman (Foundation for Science and Technology)
Dr Jeremy Gunawardena (Harvard University)
Mr Ian Hamilton (Construction Industry Computing Association)
Mr Peter Herdman (ArjoWiggins)
Prof Sir Antony Hoare (Microsoft Research)
Mr Michael Hughes (Baring Asset Management)
Dr Allan Lane (Royal Bank of Scotland)
DCI John Leicester-Finch (Thames Valley Police)
Dr Eddie Morland (AEA Technology Rail)
Prof Jim Norton (Deutsche Telekom UK)
Dr Sverrir Olafsson (BTexact Technologies)
Mr Shail Patel (Unilever)
Dr Jeff Price (Unilever)
Mr Martyn Richards (National Air Traffic Services)
Mr Matthew Roberts (RBC Capital Markets)
Sir Alan Rudge (MSI Cellular Investments)
Dr Peter Sadd (RHM Technology)
Mr Colin Sillence (BAE SYSTEMS)
Dr Andrew Smith (HBOS)
Dr Ralph Smith (formerly Jaguar Cars)
Dr Susan Stepney (Logica)
Prof Peter Stow (Rolls-Royce)
Prof Michael Walker (Vodafone)
Dr Steven Walker (Campden & Chorleywood Food Research Association)
Dr Lincoln Wallen (MathEngine)
Prof Andrew Woods (BP Institute)

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[1] Several companies supplied a small team of interviewees, in which cases the leader of the delegation is listed here.

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