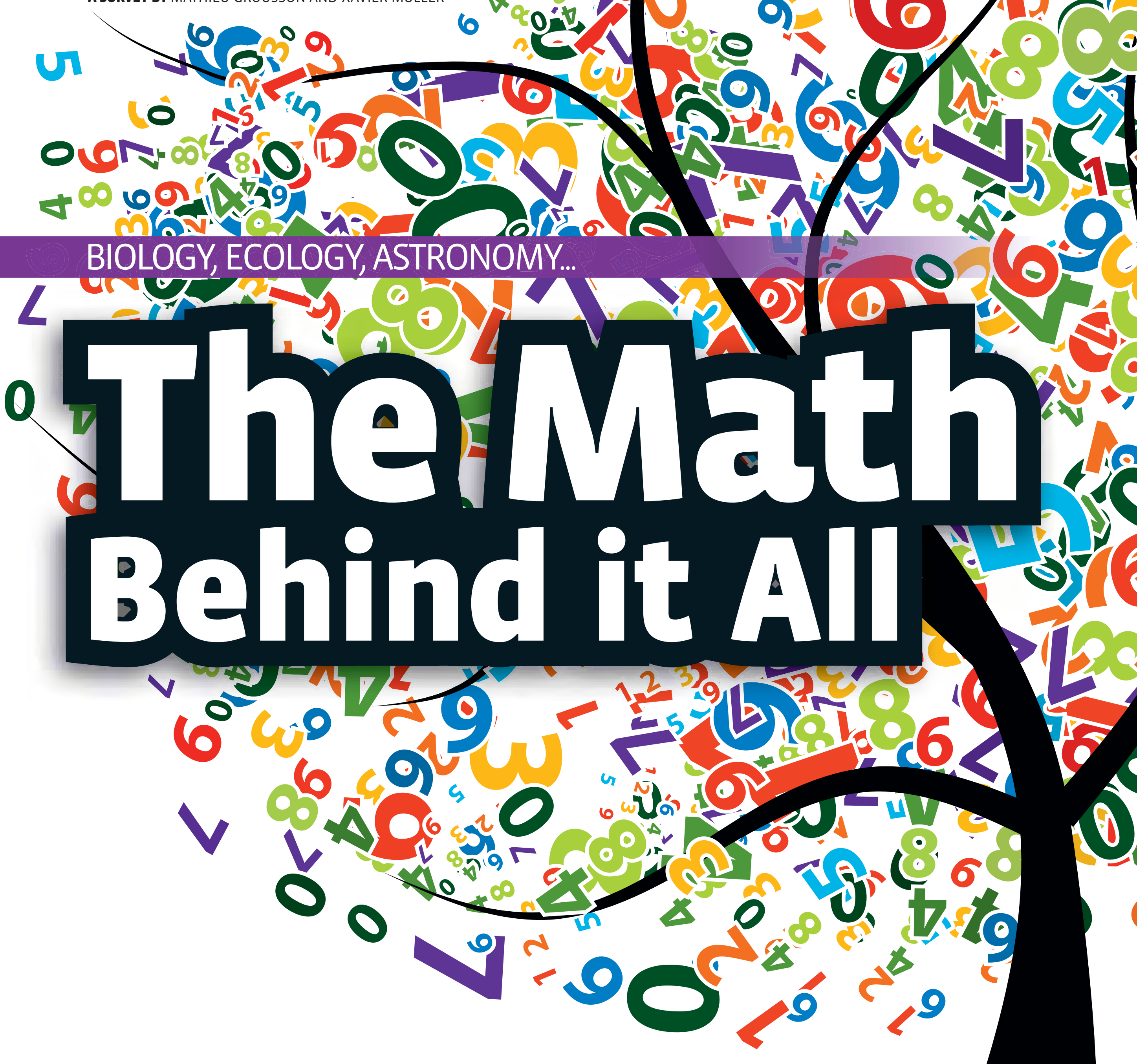
Mathematics permeates all scientific disciplines. While it has long been used to unravel the mysteries of physics, it is just as valuable for studying biodiversity, modeling epidemics, understanding brain mechanisms, or uncovering the secrets of our planet. CNRS International Magazine investigates a discipline at the heart of science.

A SURVEY BY MATHIEU GROUSSON AND XAVIER MÜLLER

BIOLOGY, ECOLOGY, ASTRONOMY...

The Math Behind it All



A Universal Discipline

When you ask him to describe the current state of mathematics, 2010 Fields Medal Laureate Cédric Villani, currently director of the Institut Henri-Poincaré[1] in Paris, answers without hesitation: "It's never been better." And the reason for this is simple: in today's high-tech world, there's hardly any human activity that does not require calculating, structuring, predicting, or optimizing increasingly complex and abundant data. To such an extent that for Amy Dahan, a science historian at the Paris-based Centre Alexandre-Koyré,[2] "today, the entire world has become the playing field of mathematics."

If math is doing so well, it is primarily due to the nature of the discipline itself and to those who work in it. The number of mathematicians has grown from a few hundred to several tens of thousands in less than a century. And since this community has always been used to working together in small, flexible, and scattered groups, it is totally at ease in a globalized world where information is exchanged at the click of a mouse.

01 The annual Mathematical Games and Culture Exhibition, in which CNRS takes part, attests to the general public's interest for the discipline.

For mathematics as such, this is a particularly fruitful time, characterized by "the emergence, with unrivalled intensity and rapidity, of new connections between different mathematical fields," explains Jean-Pierre Bourguignon, director of IHÉS,[3] a research institute centered on mathematics and theoretical physics in Bures-sur-Yvette, near Paris. Yet this does not mean that the traditional way in which the discipline is divided up (geometry, topology, algebra, analysis, and probability) has suddenly become obsolete, but rather that "just as in music, learning the classical forms makes it possible to go beyond them," as Villani explains. This is how, in 2003, the Russian mathematician Grigori Perelman managed to solve a purely topological problem, the Poincaré conjecture, with the completely unexpected contribution

01


of metric geometry. This mathematical puzzle concerning the characterization of three-dimensional spheres had been set out a century earlier by Henri Poincaré, and was one of the seven Millennium Prize Problems, whose solutions are rewarded $1 million each by the US-based Clay Mathematics Institute.

INTERTWINING RESEARCH

The constant exchanges between the various fields of mathematics are second only to its interaction with other disciplines. The most important of these is physics, which has not only been a stimulus for mathematics, but has also benefited from it for centuries. Villani believes that today still, "the greatest expectations can be found in areas where mathematics meets other disciplines." This is particularly true of partial differential equations, where major progress could have repercussions in many areas from fluid mechanics to climate research.

From there, it's only a short step to imagining that mathematics might become a universal resource. Especially as many scientific disciplines (biology, economics, sociology, or decision theory) make increasing use of sophisticated mathematical techniques and concepts. "The abstract nature of mathematics makes it a universal discipline," explains Bourguignon. "Yet it's impossible to reduce other fields to nothing else but math." Michel Morange, from the Centre Cavaillès[4] in Paris, fully agrees: "You have to be wary of statements like 'math is the future of biology.' Mathematics has a lot to contribute in a number of fields, but it would be naive to believe that it could take over other scientific disciplines."

FRANCE IN THE LEAD

Math remains one of the most thriving fields in science today. So much so that it now has followers outside specialist circles crunching numbers. Séverine Leidwanger, from the Jussieu Mathematics Institute[5] in Paris, who co-organizes events for the annual French Science Festival, has observed a renewed interest in the discipline. "Our lectures aimed at the general public attract large audiences, including people who have bad memories of math at school and suddenly realize they can understand more than they thought." Is this enough to encourage students to opt for a career in mathematics? With a tradition of making math an educational cornerstone of its elite, France remains a hub for the discipline. And it shows: 13 of the 49 laureates in the history of the Fields Medal—mathematics' most prestigious prize, awarded every four years—have come from French institutions. Yet Bourguignon is concerned about the future: "Research careers aren't attractive enough, whatever the field. The number of mathematics students is on the decline. If this trend continues over the next ten years, it could prove detrimental to the future of the discipline." Villani, on the other hand, remains optimistic: "The problems have been identified and I am confident that we will find the right solutions. The message we want to get through is that mathematics is by no means a dusty old science set in its ways. Math today is bubbling with excitement, and what it needs above all is imagination."

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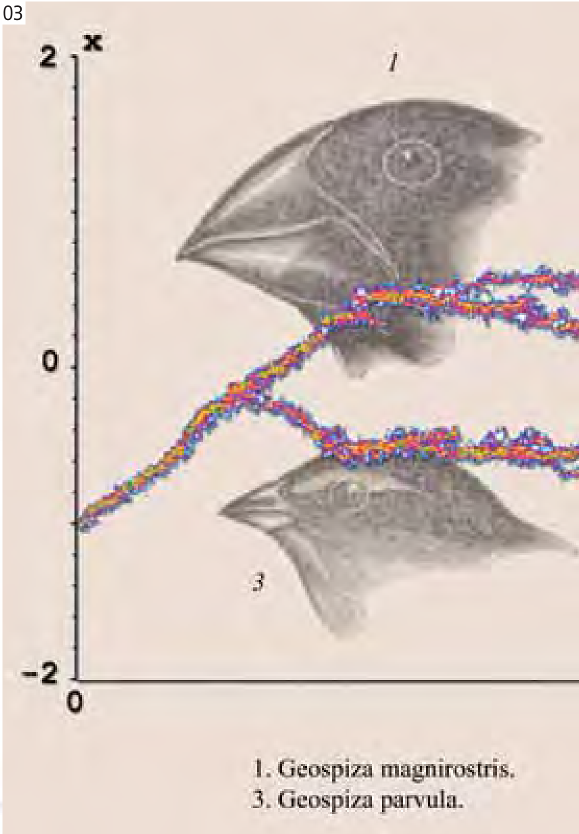
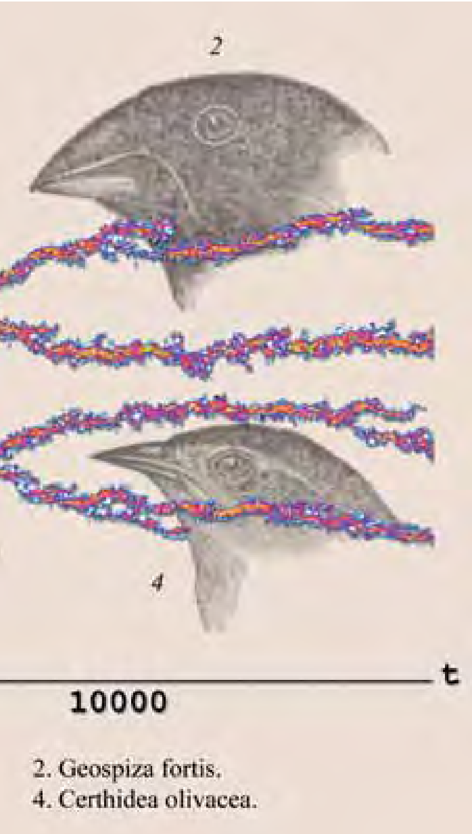


Modeling

At a time when biodiversity is declining at an unprecedented rate, ecologists now have a powerful ally in mathematics. Making use in particular of probability theory and random processes, they have succeeded in integrating major mechanisms of evolution into models. This will help quantify the biological resources of our planet and its evolution, and possibly find ways of protecting it.

Before proceeding any further, it is essential to agree on what biodiversity really means. Intuitively, it seems that the more species there are, the more abundant the biodiversity. But for Vincent Bansaye, from the CMAP,[1] things are actually more complex. "What is the contribution to biodiversity of, on the one hand, two species of ants, and on the other, an ant species and a hedgehog species?" he wonders.

The answer to this question is not as straightforward as it seems. As is the question of whether biodiversity is more



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02 Mathematics can help ecologists calculate the number of fawns in a given forest.
03 Mathematics of chance are used to model evolutionary processes, such as the emergence of chaffinch species that differ by the size of their beaks.

WHEN MATH MEETS GENETICS

Math could play a key role in understanding genetic neuromuscular diseases. Alessandra Carbone,[1] who has been involved since 2006 in the international program against muscular dystrophy, is carrying out research on the proteins associated with such conditions. "Although researchers have identified some of the genes involved in these diseases, they don't understand the precise role of the proteins they code for, which cause these dysfunctions," explains Carbone, a recent laureate of the Irène Joliot-Curie "Woman Scientist of the Year" award. In a bid to solve this problem, she has embarked on a highly ambitious research program to specify, using computer modeling, the way in which approximately 2000 proteins present in the human body interact with one another. A difficult task since two randomly chosen proteins can position themselves in relation to one another in 100,000 to 500,000 different ways. Even the hundreds of thousands of computers that make up the World Community Grid would take centuries to check all the different combinations. Researchers therefore need to limit the number of configurations to be tested. This can be done by "guessing" the location of the active sites—those with a biological function—on each protein, so as to only model parts that have a greater chance of interacting with other proteins. Of course, this is not complete guesswork.

During evolution, the active parts of proteins have often been conserved between species, and so have the DNA sequences that encode them. By coupling existing information with probability theory to process several hundreds of billions of DNA sequences stored in the world's gene banks, it is theoretically possible to identify, to a reasonable degree of probability, the conserved DNA fragments and therefore the active sites of proteins that they encode. The latter will then be tested in priority in the interaction program. "With such preparatory work, we should have finished processing the data by November 2011," Carbone reckons.

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04 Statistics applied to genome analysis, for example to identify putative chemically-active protein sites, can help accelerate research on genetic diseases.

Biodiversity

affected by the disappearance of two iconic mammal species, or by that of an obscure beetle that plays a key role in the ecosystem. And this is exactly the type of question that complex mathematics can help to answer, by introducing rigorous tools into a science that has traditionally been empirical.

Sylvie Méléard, also from the CMAP, believes that although "it would be presumptuous to say that mathematics is going to radically change the study of biodiversity, it certainly provides a fresh approach." For instance, she has recently developed a model able to describe the processes leading to the emergence of new species.[2] "To do so, we integrated an incredibly wide variety of phenomena, ranging from the genetic mutations that arise during reproduction of a single individual, to the influence of the environment on an entire population," she explains. "And by using rigorous theoretical demonstrations, we have managed to frame the conditions that lead to evolutionary steps or to the emergence of new species."

IMPROVING SPECIES INVENTORIES

Méléard, who also holds the Chair of Mathematical Modeling and Biodiversity,[3] decided to push the analysis even further by trying to determine the threshold below which a species is condemned to disappear. For her, "conventional models fail to answer this question because they are not appropriate for small populations. But by taking into account the genetic variability of individuals in a small group, we are beginning to understand how the appearance of mutations that would have had no effect in a large group, can speed up the disappearance of a small population."

Another example is the development of mathematical tools that can determine the size of a population with only partial information. Bansaye neatly sums up this problem: "Let's say I'm walking in the forest and I see a roe deer. Twenty minutes later, and about one kilometer further, I spot a second one. What significant information can I infer about the entire roe deer population from the collection of this type of data?" Such ongoing research could have a significant impact on the inventory of species that are difficult to observe. With his colleague Amaury Lambert, from the LPMA[4] in Paris, Bansaye is also studying the effect of habitat fragmentation on the evolution of populations. This work could result in drawing up better conservation measures in a context of increasingly scarce land resources allocated to wildlife.

As Denis Couvet, director of the conservation biology research unit CERSP[5] concludes: "Biodiversity can't be reduced to simple equations. But it's true that mathematical models do contribute to an impartial and objective approach to ecosystems."

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Epidemics **in Equations**

Why are diphtheria and polio a thing of the past in a number of countries? Why do we believe that malaria and tuberculosis can be eradicated from the planet? The answer, of course, lies in the existence of public health policies and in the action carried out by healthcare organizations. But however comprehensive it may be, a vaccination campaign will never stop infected individuals from slipping through. In fact, the invisible partner of vaccination campaigns is a mathematical equation that crops up in every model of infectious disease propagation. To spread, an epidemic needs a minimum number of infected people in a specific location. Below this threshold, the disease dies out—like a fire without oxygen.

HIGH-FLYING STATISTICS

In epidemiology, as this example shows, mathematics helps us understand the mechanisms of contagion. Why are certain epidemics cyclical? Why did influenza A H1N1 turn into a pandemic that rapidly spread across the planet? Without mathematics, we wouldn't have the answers to these questions. In 2008, using a mathematical model, an international team which included the theoretical physics lab LPT[1] in Orsay demonstrated that the air transport network broadly matched the map of worldwide epidemics.

And mathematics also helps in understanding the cause of epidemics of non-infectious diseases like cancer. The key discipline in this field is statistics. "The textbook case of a cancer detected statistically is lung cancer," explains Jacques Istas, a professor at the LJK[2] in Grenoble, and the author of *Mathematical Modeling in the Life Sciences* (Springer, 2005). "In the 1930s, German doctors noticed a connection between lung cancer and tobacco consumption, though they didn't understand why," says this specialist in Brownian motion—a jack-of-all-trades mathematical model that can describe such widely diverse phenomena as osteoporosis, Internet traffic, or the spread of an epidemic. Creutzfeld-Jakob disease and diabetes are among the vast group of non-contagious infections that were detected by statistical methods long before the underlying causes were discovered by biologists.

Yet just because statistics are routinely used does not mean they have broken free from their mathematical heritage. And this is fortunate, as Bernard Prum of the LSG,[3] and an expert in the genetics of epidemic diseases, explains: "Statistics are an endless source of pitfalls. Let's say that you undergo a test to find out whether you suffer from a disease that affects one person in 10,000, and this test turns out to be positive. Like all tests, it has a margin of error. Suppose this test in particular gives the right result 99 times out of 100, for both diseased and healthy people: what are the chances that you have the disease? You may think it's 99%, but you'd be mistaken. The answer is less than 1%, as calculated using Bayesan statistics." Analysis of epidemiological data is full of traps of this kind.

MONOGENIC DISEASE
A disease caused by the mutation of a single gene, as opposed to polygenic.

DISILLUSION IN NUMBERS

Now that we are able to search in the genome for aggravating factors of diseases, statistical analyses have also become far more complex. Indeed, one should bear in mind that monogenic diseases are the exception. Most diseases, especially cancers, are due to the expression of dozens of genes that need to be sought out among the 30,000 genes in the patient's genome.

"Once gene expression is taken into account, the number of parameters that need to be considered can easily reach several tens of thousands," points out Philippe Besse,[4] a specialist in the application of statistics to biology. "The difficulty lies in that it is always possible to find statistical connections between the symptoms and the genes that supposedly cause the disease, without necessarily knowing whether such connections are relevant or not." Recent research, mostly in the field of artificial intelligence, has led to the development of algorithms able to distinguish between significant connections and misleading ones. Called "boosting" or "bagging" algorithms, they allow modeling with better predictive qualities.

Besse and his colleagues are also developing mathematical tools to help biologists understand the role of specific genes and proteins in cells. Recently, the team's research has shown that bisphenol A, a chemical compound used in baby bottles and suspected of being toxic, affects the hormonal balance of mice, even when ingested at low doses.

Whether using statistics or complex algorithms, for epidemiology or for genetics, mathematicians agree that in the 21st century, the life sciences will prove as important a source of inspiration as physics was in the last century.

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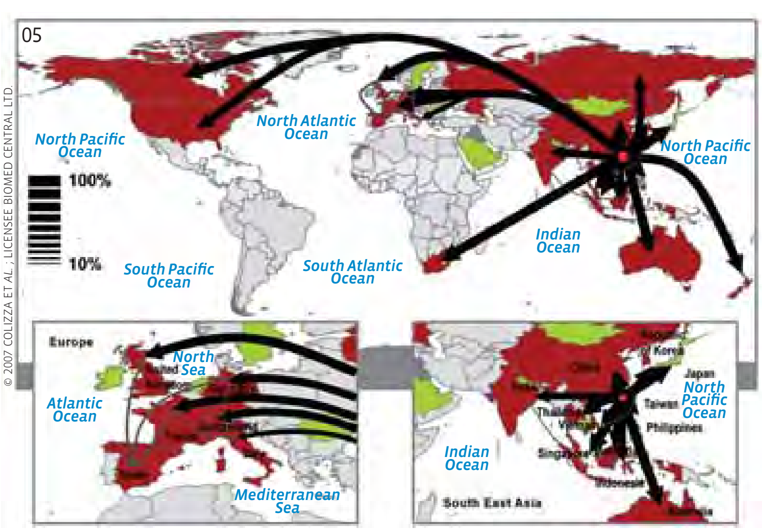
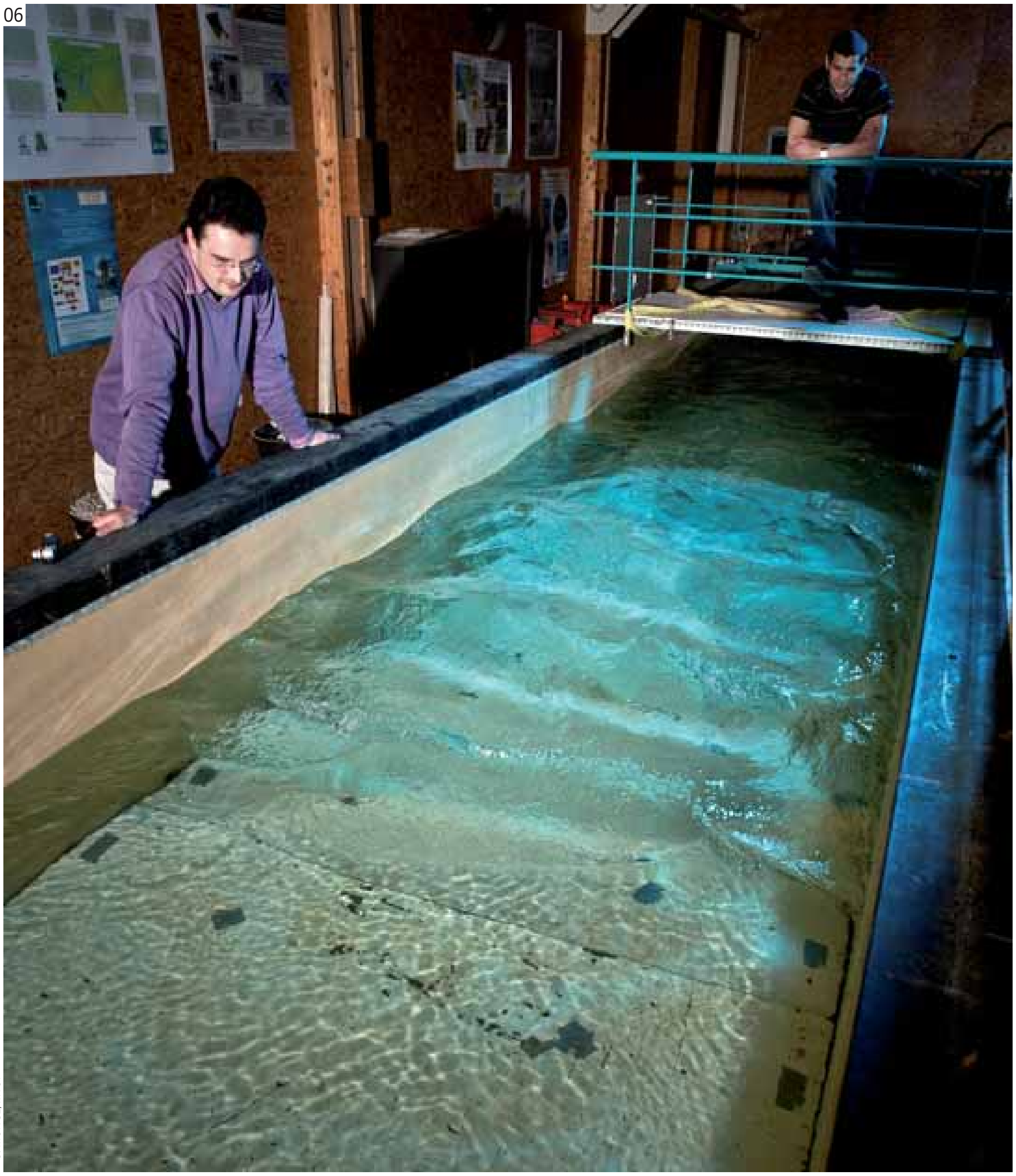


05 The computer model elaborated in Orsay uses air traffic data to predict the propagation of epidemics and pandemics, like the 2002-03 SARS outbreak, represented here. Line widths correspond to the probability of the associated path.



06 CNRS mathematicians are trying to understand the elusive astrophysical phenomenon of black holes using a wave basin.

Physics: Addicted to Math

"The book of Nature is written in the language of mathematics." The quote is from Galileo, and ever since the 17th century, numbers and abstract ideas have become inseparable from physics. So what is it that draws physicists to mathematics? It is its "unreasonable effectiveness," a statement by the 1963 Nobel Laureate in Physics, Eugene Wigner. By this, he meant its effectiveness at producing unity and fundamental laws, when mere observation only reveals disorder and the irregular nature of phenomena. Nothing could illustrate this better than a specific example that CNRS researchers have recently brought back into the limelight. In the mid-1970s, the renowned English physicist Stephen Hawking predicted that black holes, cosmic objects believed to trap even the smallest particles of light, actually emit weak radiation. Theoretical physicists were delighted, as this prediction is one of the few that, on paper, brings together general relativity (Einstein's theory of gravitation) and quantum mechanics (the realm of the infinitely small). But experimental physicists were discouraged: the intensity of Hawking radiation is so weak that no measuring device would ever be able to detect it. Yet mathematics would soon come to the rescue.

BLACK HOLE
An astronomical object so dense that nothing, not even light, can escape from it.

UNDULATING BLACK HOLES

In 1981, William Unruh, of the University of British Colombia (Canada), showed that the "ingredients" of Hawking radiation were not specific to the physics of black holes. More precisely, the same physics is obtained by replacing, on paper, gravitation by the flow of a liquid, and light waves by the ripples on the water's surface. "In this case, mathematics provided quantitative evidence of the equivalence of two apparently very different phenomena, in that they are governed by exactly the same equations," explains Germain Rousseaux, of the LJAD[1] in Nice. Indeed, Rousseaux has recently set out to detect Hawking radiation in a wave basin belonging to the Nice-based company ACRI, which specializes in coastal engineering. In this experiment, Hawking radiation should take the form of ripples produced in an area of the basin where the propagation of a wave is stopped dead in its tracks by a current flowing in the opposite direction.

THROUGH THE MAGNIFYING GLASS

For this specific application, mathematics will also be used to sift through the large amount of data generated by the experiment. "We have included in the theoretical description of our study the mathematical tools of dynamic systems, which are traditionally used to study the evolution over time of systems such as planets of the Solar System," Rousseaux explains. "This tends to show that the

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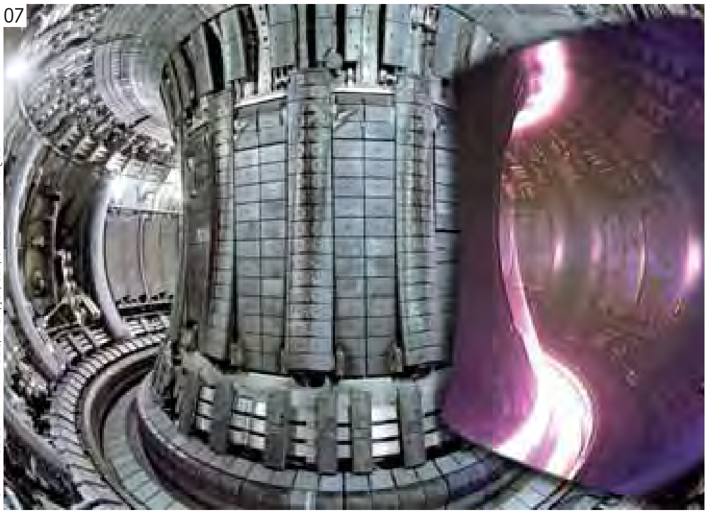

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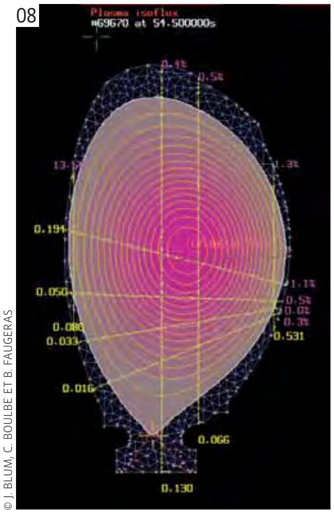

07 08 The JET reactor (left) in Culham (UK), makes it possible to study thermonuclear fusion. Its combustion chamber contains a plasma (pink), whose complete characteristics are calculated using the Equinox software (right).

more you resort to complex mathematics to study a phenomenon, the more details you reveal. Math acts like a magnifying glass for physics."

In some cases, math may even be the only way of gaining access to certain phenomena—like those that will take place in the future International Thermonuclear Experimental Reactor (ITER), currently under construction in Cadarache, France. ITER is a prototype designed to verify the feasibility of nuclear fusion as a new source of energy. In the reactor, an ionized gas confined by a powerful magnetic field will be heated to a temperature of around 100 million °C, which no sensor could ever withstand. The only solution is to collect data measured indirectly at the surface of the plasma, and incorporate it into a mathematical model to perform a numerical computation of what's happening inside. "To do this, it was necessary to develop an approximation of the equation to be solved," explains Jacques Blum, also from the LJAD. "Here, the role of mathematicians is to verify the extent to which the equation remains valid for studying the initial problem. Besides, with ITER, we want to apprehend the internal parameters of the plasma in real time, to be able to continuously monitor the experiment. This makes it necessary to develop new algorithms that are both fast and reliable."

INVENTING NEW TECHNIQUES

Admittedly, the equations that govern the ITER plasma have long been known, and come from the field of hydrodynamics. Yet new mathematical techniques constantly need to be invented to solve these complex equations in the conditions that will prevail in the ITER facility. "This requires the development of genuine mathematical engineering," explains Pierre Degond, a researcher at the Toulouse Mathematics Institute,[2] who is also involved in the ITER program. "It is necessary to deal with the complex interactions between different spatial and temporal scales, from that of an atom to the entire plasma." Today, philosophers of science still question the real significance of Galileo's words. But one thing is certain: if it weren't for mathematics, physics as we know it simply wouldn't exist.

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09 Complex and sophisticated mathematics plays a key role in determining the best trajectory of a rocket (in this case Ariane 5).

HELPING ARIANE FIND ITS WAY

Surprisingly, the equations used to launch rockets have been known since the 17th century. But today's space industry resorts to cutting-edge mathematics, in particular to calculate trajectories precisely. The launch of the European Ariane rocket involves a French collaboration between the mathematics research hub MAPMO[1] in Orléans, the University of Orléans, and EADS Astrium in Les Mureaux. "The software used to calculate the optimal trajectory of an Ariane launcher has to compute hundreds of trajectories before selecting the best one. This process requires a huge amount of computing time," explains Emmanuel Trélat, who is currently tackling this problem with Thomas Haberkorn at the MAPMO.

When weather conditions change, for example, a launch may be postponed by a few days. To be able to adapt in real time to unforeseen events, Trélat is developing new software capable of making a preliminary selection of possible trajectories, before fully calculating the parameters of the chosen solution. "The principle underlying this method dates from the 1950s," indicates Trélat. "But it requires a great deal of expertise to implement it in specific situations." The new software should be operational in late 2011.

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Math in the Brain

As biologists know only too well, mathematics has become an essential tool to explore and understand the brain's complex mechanisms. "Not only does the brain contain roughly 100 billion neurons of various types," explains Pascal Chossat, a mathematician at the LJAD,[1] "but in addition, these neurons are closely interconnected." On average, each neuron links to 10,000 other neurons. They form circuits with highly variable structures, and connections that change over time. Besides, dynamic aspects are essential. For instance, the electrical signals that enable neuron inter-communication can either reinforce or interfere with one another.

Biologists, with the help of mathematicians, are working flat out to decipher this complexity. "The application of mathematics to brain research is booming," adds Chossat, member of a team of researchers currently using mathematics to understand the visual network, a project led by Olivier Faugeras of the LIENS.[2] In fact, the visual network perfectly mirrors the tree-like structure of the brain: according to their location on the retina, nerve impulses from the eyes are dispatched to various regions of the visual cortex which are themselves subdivided into areas that specialize in a specific task, like detecting a color or a shape.

Chossat, a specialist in bifurcation theory, a discipline that aims to describe sudden changes in the behavior of systems, is seeking to apply his knowledge to the visual hallucinations experienced by schizophrenics as well as by people under the influence of certain drugs like LSD. "We want to understand how an image spontaneously appears in the visual cortex," Chossat says. His research has revealed that some mechanisms at work in the visual cortex can be represented by functions defined in a hyperbolic space, a strange type of space where two lines parallel to a third can in fact intersect. "This is the first time that this type of object has cropped up in biology," Chossat enthuses.

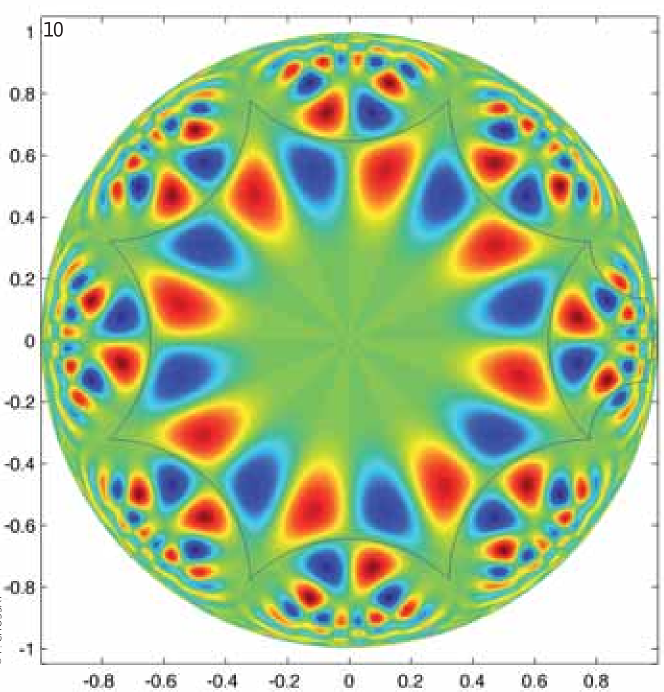
IMAGING AID

If mathematics plays an increasingly important role in neurobiology, it is partly due to the imaging techniques used to see neurons in action. These techniques, based on the recording of the particles emitted in the brain, are highly dependent on applied mathematics.

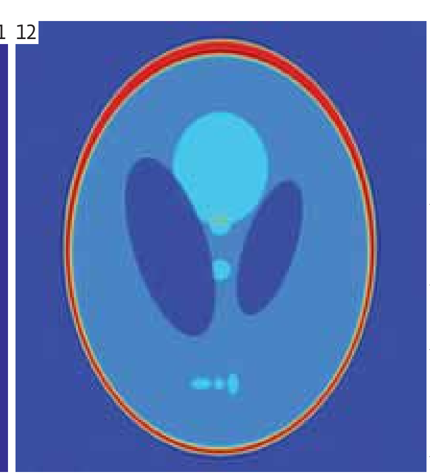
To determine which regions of the brain emit the particles, the machines use a mathematical operation called inversion—which basically reverses their paths to the source. Seismologists use the same kind of mathematical tool when they trace seismic waves back to the source of an earthquake.

IMAGING TECHNIQUES

Electroencephalograms and magnetic resonance imaging (MRI) detect photons, while positron emission tomography (PET) focuses on protons, and so on.



10 Mathematical representation of a type of brain activity that can occur spontaneously and provoke hallucinations.



11 12 Numerical simulation of photoacoustic imaging performed on a Shepp-Logan phantom, a well-known representation of the human brain widely used to test medical imaging techniques (original (left) and reconstructed (right) images).

COMPLEXITY IN OUR VEINS

By the time you have read this sentence, your bone marrow will have produced several million blood cells. This phenomenon, known as hematopoiesis, is an incredibly productive and complex machinery, involving thousands of factors such as genes, proteins, or hormones. This is why biologists have trouble understanding hematopoietic dysfunction, which may lead to leukemia or chronic anemia (a shortage of red blood cells). To help them get a clearer picture, Vitaly Volpert and his colleagues at the Institut Camille-Jordan[1] in Villeurbanne are developing models where bone marrow cells are represented as tiny spheres within the extracellular matrix, which is regarded as being a uniform environment where molecules can diffuse and affect cells. Their research has already shown that a single type of imbalance in this complex system can cause a host of disorders, which may in addition vary considerably from one individual to another. No wonder doctors find it so hard to establish a precise diagnosis. "Currently, we are still trying to understand these underlying mechanisms," says Volpert. "But in the long term, we hope that this model will help cure diseases and develop customized treatment for each patient."

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The images obtained through various techniques like MRI or magnetoencephalography also require sophisticated mathematics to be interpreted. Is this white patch on an MRI a benign or a malignant tumor? The image alone cannot give the answer. "The problem is that MRIs give no information on physical parameters like hardness or electrical conductivity that can point to a tumor," says Habib Ammari of the DMA-ENS,[3] who is developing mathematical and numerical models for new medical imaging systems. This is true of a wide range of other imaging techniques. To help doctors make a diagnosis, mathematical models of the response of brain tissue to imaging methods have therefore been developed. These models help anticipate how diseased regions will look like on the actual images.

Another way to improve diagnoses is emerging, namely the combination of different imaging techniques. Together with teams led by Mathias Fink and Claude Boccara, both physicists at the Institut Langevin[4] in Paris, Ammari and his group have developed new imaging methods that combine the use of sound waves (as in ultrasound scans) and electromagnetic waves. When applied to breast cancer, these methods show excellent selectivity, giving virtually no false positives and providing incomparably sharp images. In the future, similar combinations of multi-wave techniques could very well bring about radical changes in medical imaging. Since reconstructing images is a highly complex undertaking, mathematicians will play a key role in this revolution.

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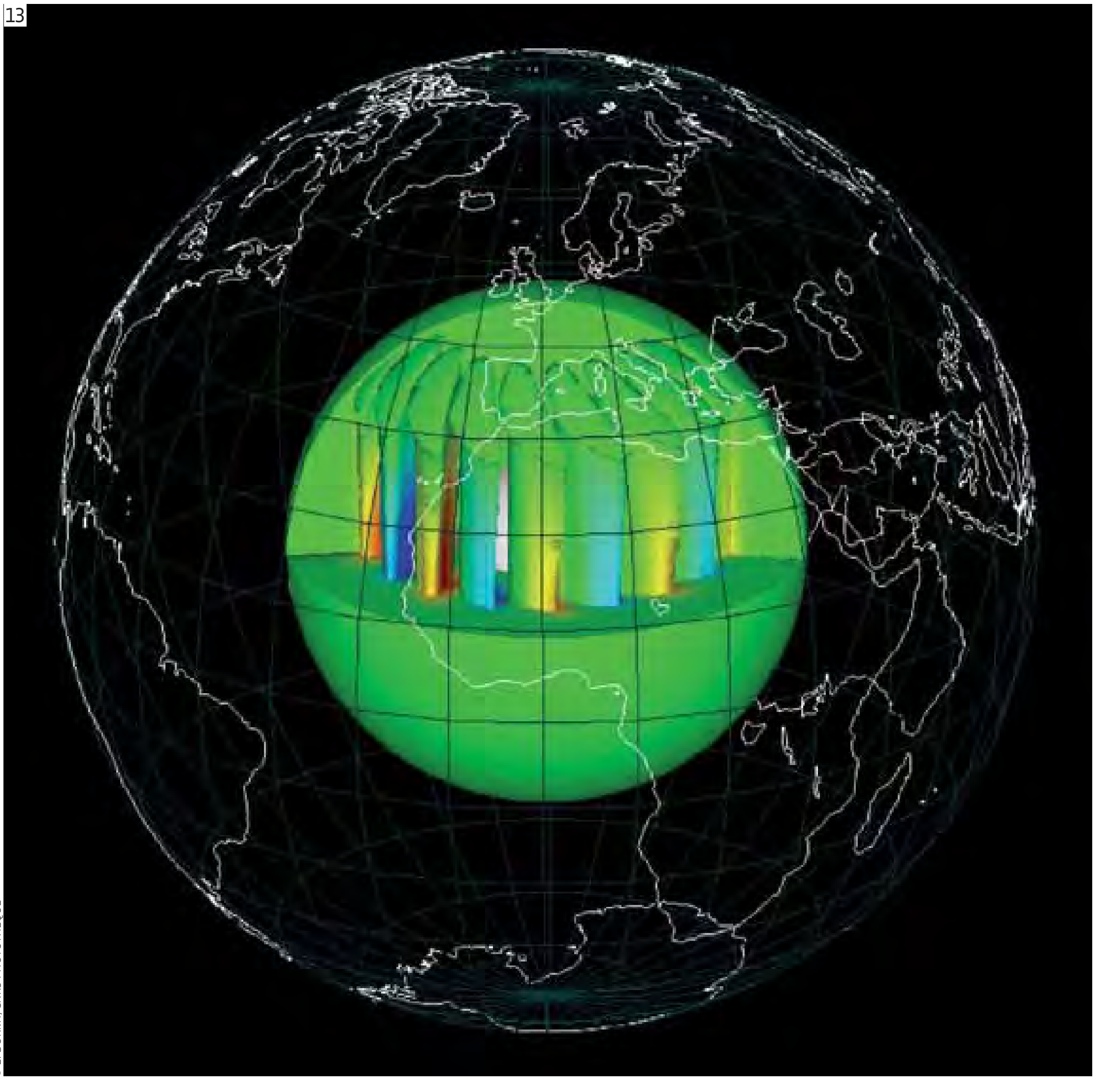
Tuned into Earth

In Earth sciences as in other fields, mathematics is key to helping researchers untangle the complexities of our planet, as shown in the following examples.

The first one concerns tsunamis, a phenomenon that regularly affects the wellbeing—and often survival—of populations of the Pacific and Indian Oceans. How do these deadly waves form, and how do they travel across the ocean? Is it possible to prevent the flooding they cause? In an effort to answer these questions, several tsunami prevention groups throughout the world use hydrodynamic numerical models. But classical models frequently get bogged down in the calculations, and are incapable of dealing with atypical situations, such as extremely jagged coastlines or highly uneven seabeds.

The VOLNA numerical model developed in 2008 by Denys Dutykh and his colleagues at the Mathematics Laboratory[1] of the University of Savoie has no such drawbacks. "Our model uses the latest advances in numerical computing, which had never been used before in this field," Dutykh explains. This model is also able to correctly reproduce tsunamis that propagate across the ocean while the sea floor is still active, like the one that struck the Indonesian island of Sumatra in 2004. It was caused by an earthquake that lasted 10 minutes and led to an extremely complex situation in



13 Diagram of the thermal convection movements that take place within the Earth's core.

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terms of propagation. But earthquakes are not the only natural disasters that generate tsunamis. The team studied the case of submarine landslides in the Saint Lawrence River in Quebec (Canada). Such landslides give rise to high waves that then sweep onshore, flooding homes. The researchers were able to use their expertise to produce flood maps for the areas at risk.

INVALUABLE ALGORITHMS

Water can also be devastating in landlocked areas. Although the consequences are less dramatic, the runoff of rainwater from fields is a real scourge for farmers and local residents. Runoff can wash up to several tens of tons of earth down-

14 Map of the Saint Lawrence riverbed, in Canada. This data was used by Denys Dutykh to study flood risks resulting from river landslides.

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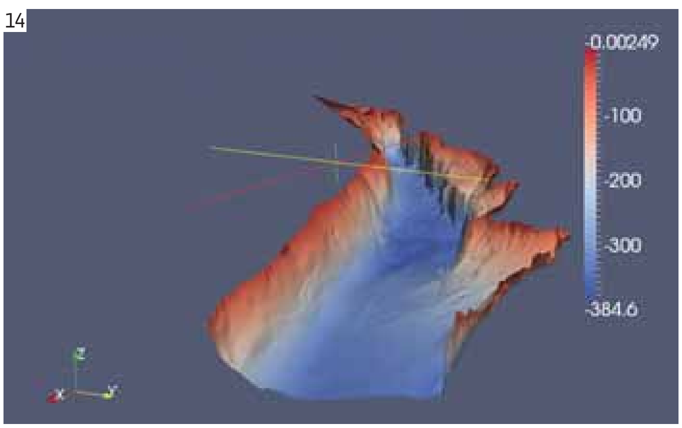

stream per year and per hectare, leading to a significant fall in crop yields, or causing flooding and mudslides. A number of anti-erosion measures have been taken in France over the past few years, such as planting vegetation during cropless periods—so that the ground is not left bare—or restoring hedgerows. But are such measures really effective? That's what the METHODE project, which brings together hydrologists, mathematicians, agronomists, and computer scientists, is trying to find out by developing novel numerical models of surface runoff.

"What's difficult about simulating the flow of water across a plot of land is that the sheet of water has a thickness similar to that of the imperfections of the ground, basically a few centimeters," explains Cédric Legout, of LTHE[2] in Grenoble. The presence of small-scale turbulence, the separation of the flow into pools, and the flooding of furrows in the ground make it difficult to predict the total runoff from the plot. It is up to the group's physicists to simplify these phenomena, and identify those that play a leading role on a given scale. "The mathematicians then have to ensure that the algorithms are compatible with the physicists' simplifications," Legout adds. Scientists hope that such research could make planning policies more efficient in the future, thus protecting people and crops from water's devastating effects.

AVOIDING OVERSIMPLIFICATION

It is not always possible to strip a physical system down to its bare essentials. This is true for the convective motions within the Earth's core, which produce the terrestrial magnetic field through a self-excited dynamo effect. The origin of the field and some of its properties—it reverses approximately every 100,000 years—are still debated. "We have a series of mathematical theorems that show that if the models of the terrestrial dynamo are oversimplified, these mysteries become impossible to solve," explains Emmanuel Dormy of IPGP/ENS.[3]

The research group he belongs to studies phenomena such as ocean and atmospheric circulation, or the terrestrial dynamo which involve flows of matter on different scales, but that have a mutual effect on one another (e.g., ocean currents or winds). The researchers are trying to identify which of the processes involved can be simplified without losing their essential features. Since they can't tackle the geodynamo problem in its entirety, the team has focused on the friction between the flow of liquid metal and the core-mantle boundary, roughly 3000 km below the surface. By doing so, they have reached the counterintuitive conclusion that the friction is less intense on this rough surface than if the same surface had been smooth. It's by stacking up this kind of data that the geodynamo will eventually be deciphered. As the famous saying goes, "the devil is in the details." So is mathematics.

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02. Laboratoire d'étude des transferts en hydrologie et environnement (CNRS / Université Grenoble-I / Institut polytechnique de Grenoble / IRD).
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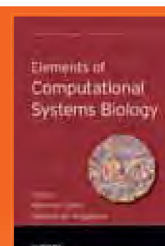

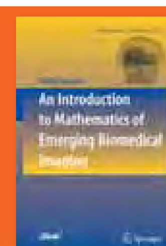

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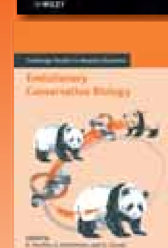

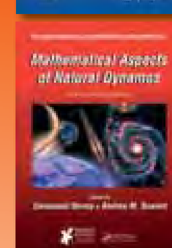

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