OPPORTUNITIES FOR THE
MATHEMATICAL SCIENCES
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Getting Inspiration from Electrical Engineering and Computer Graphics to Develop Interesting New Mathematics

I. DAUBECHIES

Applied mathematicians can use their craft to help scientists and engineers solve numerical, simulational, or modeling problems, and they have often done so, in many fields, with great success, developing new applied mathematics tools as they progress.

Talking to scientists and engineers should also lead to forms of applied mathematics where the science or engineering problems pose conceptual challenges that force us to (hopefully) develop whole new fields of mathematics (ultimately). It is much harder to foretell what these fields will look like--the power and importance of Fourier Analysis would have been hard to predict before Fourier. Our only hope to not miss these chances is to work hard and earnestly with scientists and engineers to get steeped in their problems, and not just be "consultants" for them. But it can be very hard to do this.

Below are two "case studies," where we (my collaborators and myself) did not succeed in creating new fields, but where we tried very hard to listen to the real concerns of engineers while formulating approaches.

1. A/D CONVERSION IN ELECTRICAL ENGINEERING

Digital signal processing has revolutionized the storage and transmission of audio signals, images and video, in consumer electronics as well as in more scientific settings (such as medical imaging). The main advantage of digital signal processing is its robustness. Although all operations have to be implemented with necessarily not quite ideal hardware, the *a priori* knowledge that all ideal outcomes must lie in a very restricted set of well separated numbers makes it possible to recover the ideal outcomes by rounding off appropriately. When bursty errors can compromise this scenario (as is the case in many communications channels, as well as for storage in memory), making the "perfect" data unrecoverable, knowledge of the type of expected contamination can be used to protect the data, prior to transmission or storage, by encoding them with error correcting codes. This is again done entirely in the digital domain. All these advantages have contributed to the present widespread use of digital signal processing. Many signals, however, are inherently "analog" rather than digital in nature; audio signals, for instance, correspond to functions, modeling rapid pressure oscillations, which depend upon a "continuous" variable, and the range of the signal typically also fills an interval. For this reason, the first step in any digital processing of such signals must consist of a conversion of the Analog signal to the Digital world, usually abbreviated as A/D conversion. Note that at the other end of the chain, after the signal has been processed, stored, retrieved, transmitted, ..., all in digital form, it needs to be reconverted to an analog signal that can be understood by a human hearing system; we thus need a D/A conversion there.

The digitization of an audio signal rests on two pillars: sampling and quantization.

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It is standard to model audio signals on bandlimited functions, i.e., functions for which the Fourier transform vanishes outside an interval $[-\Omega,\Omega]$. For such functions one can use a well-known sampling theorem that an Ω-bandlimited function is completely characterized by sampling it at the corresponding Nyquist frequency Ω/π. However, this is not useful in practice: if the samples are not known perfectly, and are replaced by noisy versions, then stable reconstruction is no longer possible. To circumvent this, it is useful to introduce oversampling, which buys the freedom of using more local reconstruction formulas that can stably reconstruct an approximation of the signal if noise contaminates the samples. In practice, it is customary to sample audio signals at a rate that is at least 10% or 20% higher than the Nyquist rate. For high quality audio, a traditional sampling rate is 44,000 Hz.

The above discussion shows that moving from "analog time" to "discrete time" can be done without any problems or serious loss of information. At this state, each of these samples is still a real number. The transition to a discrete representation for each sample is called quantization.

The simplest way to "quantize" the samples would seem to replace each by a truncated binary expansion. If we can "spend" k bits per sample, then a natural solution is to just select the first k bits in the binary expansion of the sample value. Quantized representations of this type are used for the digital representations of audio signals, but they are, in fact, not the solution of choice for the A/D conversion step. (Instead, they are used after the A/D conversion, once one is firmly in the digital world.) The main reason is that it is very hard (and therefore very costly) to build analog devices that can divide the amplitude range into 2^{k+1} precisely equal bins. It turns out that it is much easier (=cheaper) to increase the oversampling rate, and to spend fewer bits on each approximate representation. Sigma-Delta quantization schemes are a very popular way to do exactly this: they oversample significantly, and then spend very few bits per sample, but nevertheless achieve a very close approximation for the overall function when coarsely quantized "samples" are used instead of the true samples. In the most extreme case, every sample is replaced by just one bit. Surprisingly, very little mathematical work has been done of these systems. In recent work with R. DeVore, C. Gühtürk and O. Yilmaz, many interesting results were obtained, but we also constantly had to go back to the drawing board as we understood better what the engineers meant. The process of keeping in touch with engineers at all stages was essential.

2. COMPUTER GRAPHICS

Computer graphics will become increasingly important in visualization not only of little animated characters, but also of complex scientific data and simulations. Already subdivision surfaces do a better job in numerical computations for thin shell models than the standard finite elements (Ortiz and Schröder). There are many challenges for mathematicians in computer graphics, with too few mathematicians interested in them at the moment. Insights in analysis, differential geometry and topology, as well as complexity analysis, are all needed, in order to understand how best to represent, compress, evolve data on surfaces.

3. UNDERSTANDING THE INTERNET

This was the topic of one of three parts of an NSF workshop on interactions between mathematics and computer science. The problems exposed by Internet researchers clearly needed expertise from graph theorists, from topologists, from statisticians, from geometers, from PDE experts, and probably many more. One of the challenges of such areas is to get enough people from these groups interested, not to just write a paper from their point of view, but to really work with others.

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