OPPORTUNITIES FOR THE MATHEMATICAL SCIENCES

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The Year 2000 in Geometry and Topology

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I will start with a preamble to introduce a definition for the science of mathematics as to distinguish it from the physical sciences.

Mathematics consists of the study of all possible worlds, with the goal of uncovering transcendent, universal relationships and underlying symmetries.

By way of contrast, fields such as physics or chemistry or biology are concerned, by definition, with the details of the particular universe that we inhabit. This is to say that the charge for the physical sciences can be summarized as follows:

Provide a predictive understanding of the given universe.

The physical sciences find mathematics useful, and often remarkably so, because the underlying relationships which transcend our particular universe yield predictions which would be unfathomable with a focus that restricts solely to our own world.

In any event, abstract mathematics research, understood as the exploration of the contingent possibilities for a universe, is as important a scientific undertaking as the exploration of the universe at hand. Utility to other sciences only strengthens the import of pure mathematical research. However, an honest comparison of abstract mathematics to other scientific enterprises must mention one difference: Lucrative and patentable discoveries in pure mathematics are relatively rare.

The preamble is now over, and what follows is a brief description of three areas of geometry and topology that will see, to my thinking, some very exciting developments in the next few years.

The first area concerns three- and four-dimensional differential topology. At issue here is the complete classification of all three- and, likewise, four-dimensional spaces which look to a local observer to be smooth and flat. In this regard, I will tie into my preamble by saying that these subjects are quite explicitly about listing all possible shapes for the universe (with `time' as the fourth dimension). Of course, the precise shape of our particular universe is a question for astrophysics and cosmology: the mathematical goal is to determine the list of possible shapes.

For the three-dimensional case, there is a very precise conjecture (known as the Geometrization Conjecture) which postulates that any three-dimensional shape of the type under consideration can be made in a modular fashion using copies from a set of eight extremely symmetric homogeneous shapes. This conjecture is almost universally believed to be true, and there are no known potential counterexamples. It is entirely possible that some or all of the as yet unproved subcases of the conjecture will be established in the next few years. A complete proof of the conjecture would close the book on a fundamental question that has been under intense investigation since the first decade of the twentieth century.

By way of contrast, the four-dimensional classification problem is wide open; there are no reasonable conjectures for the answer. However, recent new tools have been brought to the subject. These include new differential equations coming from quantum physics and also from classical complex number theory. These novel tools have led to the demolition of all of the old conjectures. Moreover, the full power of these new tools has yet to be determined; so the next few years may be

and time

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quite exciting as their power is more fully explored.

The second area of excitement centers on a subject called "mirror symmetry." This subject concerns a very unexpected and unexplained symmetry which relates some special six-dimensional spaces that arise in string theoretic physics. To bring my preamble into play, you should note that only a very small set of these spaces could provide a reasonable string theoretic model for the real world; the rest presumably describe unrealized string theoretic universes.

The special six-dimensional spaces are known as Calabi-Yau manifolds and have been studied by pure mathematicians independently for quite some time. However, their applications to string theoretic physics led to the discovery of a sort of transformation amongst these spaces which relates the properties of one to some very different properties of another. It is as if the points in one Calabi-Yau space correspond somehow to certain three-dimensional subspaces of another. Corresponding Calabi-Yau spaces are called mirrors of each other, hence the name "mirror symmetry." In any event, this mirror symmetry implies the most unexpected relationships between seemingly unrelated properties of seemingly unrelated spaces. This sort of symmetry is completely new to mathematics and has generated a huge amount of interest and research. Here, the research touches on a very diverse suite of mathematics, ranging from differential equations to algebra and number theory.

By the way, the full scope of the implications of the mirror symmetry has yet to be determined. Moreover, there is some speculation that a version of mirror symmetry may exist for a more general set of spaces.

My final area to discuss is that of complex dynamics. The goal here is to understand the appearance of chaos in a suite of highly idealized models for nonlinear dynamics. To tie the story to my preamble, let me remind you that many of nature's phenomena have chaotic, turbulent or fractal-like properties. Meanwhile, the underlying equations of motion are universally believed to be completely deterministic. (Even quantum mechanics is deterministic in an appropriate sense.) However, our ability to predict the appearance and form of chaos from the underlying deterministic equations is at a very rudimentary stage. Thus, the study of idealized dynamical systems can shed light on the inevitably messy dynamics of the real world.

In any event, the subject of complex dynamics concerns some very special dynamics on a spherical or planar surface. The dynamics here can be chaotic in some regions and completely controlled in others. Most probably, you have seen pictures of very complicated "Mandelbrot sets;" the understanding of these sets is part of what is at stake here. More to the point, the goal is to develop a predictive understanding for the appearance and type of chaos which can arise in these dynamical systems. The techniques involve a suite of ideas which range from the theory of complex numbers through differential equations to "renormalization" as practiced by physicists. In recent years, a conjecture of sorts has crystallized which, if true, provides a fundamental basis for our understanding of chaotic phenomena.

The verification of this conjecture is certainly within reach and may well occur in the next few years. The field now is very exciting, as a host of top mathematicians are involved in the investigations. By the way, it is also the case that there are intriguing parallels between the phenomena which appear here and those which appear in the study of three-dimensional manifolds. The elucidation of these parallels has been beneficial to both subjects.

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